A GLANCE ON MAG - THIRU INTEGER SEQUENCE

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Abstract:

A new integer sequence called Mag-Thiru integer sequence generated from the recurrence relation $MT_{n+2} = 9\,MT_{n+1} - 14MT_n$, $n \geq 0$ with the initial conditions $MT_0=9$, $MT_1=53$ is analyzed for varieties of interesting properties.

Keywords : Integer sequence , Mag-Thiru numbers

Notations :

Mersenne number $M_n = 2^{n+1} - 1$, $n = 0,1,2,...$

Mag-Shanthi number $MS_n = 7^{n+1} - 1$, $n = 0,1,2,...$

Kynea number $Ky_n = (2^n + 1)^2 - 2$, $n = 0,1,2,...$

Carol number $Carl_n = (2^n - 1)^2 - 2$, $n = 1,2,3,...$

Thabit ibn Qurrah number $Th_n = 3*2^n - 1$, $n = 0,1,2,...$

1.Introduction

Number is the essence of mathematical calculations. Numbers have varieties of patterns and have varieties of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on. In this context one may refer [1-13]. This communication presents varieties of fascinating properties on Mag-Thiru numbers.

2.Method of analysis

The Mag-Thiru sequence denoted by $\{MT_n\}$ is defined by the

recurrence relation

$$MT_{n+2} = 9\,MT_{n+1} - 14\,MT_n \tag{1}$$

with the initial conditions

$$MT_0 = 9, MT_1 = 53 \quad (2)$$

The auxiliary equation associated with the recurrence relation (1) is given by

$$m^2 - 9m + 14 = 0 \quad (3)$$

whose roots are

$$m_1 = 2, m_2 = 7$$

Thus, the general solution of (1) is

$$MT_n = A*2^n + B*7^n$$

From the initial conditions ,we infer that

$$A = 2, B = 7$$

Hence, the n^{th} integer in the sequence $\{MT_n\}$ is given by

$$MT_n = 2^{n+1} + 7^{n+1}$$

A few Mag-Thiru numbers are given by

9, 53,351,2417,16839,117713,…

A few interesting properties among Mag-Thiru numbers are presented below :

Property 1: Note that Mag-Thiru numbers are odd numbers and the triple

$$[MT_n, \frac{(MT_n)^2 - 1}{2}, \frac{(MT_n)^2 + 1}{2}]$$ is a Pythagorean triple.

Property 2: $14\,MT_{n-1} = 7\,M_n + 2\,MS_n + 9$

Proof

By definition

$$MT_{n-1} = 2^n + 7^n$$

Therefore

$$\begin{aligned} 14\,MT_{n-1} &= 7*2^{n+1} + 2*7^{n+1} = 7*(M_n + 1) + 2*(MS_n + 1) \\ &= 7*M_n + 2*MS_n + 9 \end{aligned}$$

Property 3: $MT_{2n+1} - MS_{2n+1} - 1 = (M_n + 1)^2$

Proof

By definition

$$MT_n = 2^{n+1} + 7^{n+1} = 2^{n+1} + MS_n + 1$$

Replacing n by $2n+1$ in the above equation , we have

$$MT_{2n+1} - MS_{2n+1} - 1 = 2^{2n+2} = (M_n + 1)^2 \quad (4)$$

Property 4: $MT_{2n+1} - MS_{2n+1} = 4Ky_n - 4M_n + 1$

Proof

Consider (4) of Property 3.

$$4 * 2^{2n} = 4[(2^n + 1)^2 - 2^{n+1} - 1]$$

R.H.S. of (4) =
$$\begin{aligned} &= 4[(2^n + 1)^2 - 2 - 2^{n+1} + 1] \\ &= 4[((2^n + 1)^2 - 2) - (2^{n+1} - 1)] \\ &= 4(Ky_n - M_n) \end{aligned}$$

Hence the property.

Property 5: $MT_{2n+1} - MS_{2n+1} = 4M_n + 4Carl_n + 9$

Proof

Consider (4) of Property 3.

$$4 * 2^{2n} = 4[(2^n - 1)^2 + 2^{n+1} - 1]$$

R.H.S. of (4) =
$$\begin{aligned} &= 4[((2^n - 1)^2 - 2) + (2^{n+1} + 1)] \\ &= 4[((2^n - 1)^2 - 2) + (2^{n+1} - 1) + 2] \\ &= 4(Carl_n + M_n) + 9 \end{aligned}$$

Hence the property.

Property 6: $3(MT_{2n+1} - MS_{2n+1} - 1) = 4(TH_{2n} + 1)$

Proof

Consider (4) of Property 3.

$$3*2^{2n+2}=12*2^{2n}$$

$$3 * \text{L.H.S. of (4)} = \quad = 4[(3*2^{2n}-1)+1]$$
$$=4(TH_{2n}+1)$$

Property 7: $\sum_{i=0}^{n-1} MT_i = \frac{MS_n + 6M_n - 12}{6}$

Proof

$$\text{L.H.S.} = \sum_{i=1}^{n}(7^{i+1}-1) = 7\sum_{i=1}^{n}7^i - n = \frac{7(7^{n+1}-7)}{6} - n$$
$$= \frac{7(7^{n+1}-1-6)}{6} - n = \frac{7MS_n}{6} - 7 - n = \text{R.H.S.}$$

Prpperty 8: $\sum_{i=0}^{n-1}(MT_i)^2 = \frac{4}{3}M_{2n-1} + \frac{49}{48}MS_{2n-1} + \frac{28}{13}[M_{n-1} * MS_{n-1} + M_{n-1} + MS_{n-1}]$

Proof

$$\text{L.H.S.} = \sum_{i=0}^{n-1}(2^{i+1}+7^{i+1})^2$$
$$= 4\sum_{i=0}^{n-1}2^{2i} + 49\sum_{i=0}^{n-1}7^{2i} + 28\sum_{i=0}^{n-1}14^i$$
$$= \frac{4(2^{2n}-1)}{3} + \frac{49(7^{2n}-1)}{48} + \frac{28(14^n-1)}{13}$$
$$= \frac{4\,M_{2n-1}}{3} + \frac{49\,MS_{2n-1}}{48} + \frac{28}{13}[(2^n-1)(7^n-1)+2^n+7^n-2]$$
$$= \frac{4\,M_{2n-1}}{3} + \frac{49\,MS_{2n-1}}{48} + \frac{28}{13}[(2^n-1)(7^n-1)+(2^n-1)+(7^n-1)]$$
$$= \text{R.H.S.}$$

Property 9

$$(MT_n - M_n - 2)\,[\sum_{i=1}^{k}7^i\,(MT_{(i+1)n} - M_{(i+1)n} - 1)] = (MT_n - M_n - 1)^2\,[(MT_n - M_n - 1)^k - 1]$$

Proof

By definition, one has

$$MT_n - M_n - 1 = 7^{n+1} = 7*7^n \tag{5}$$

Replacing n by 2 n in (5) , we have

$$MT_{2n} - M_{2n} - 1 = 7*7^{2n} \quad (6)$$

From (5) & (6) ,one obtains after some algebra that

$$7(MT_{2n} - M_{2n} - 1) = (MT_n - M_n - 1)^2 \quad (7)$$

Replacing n by 3 n ,4 n ,5 n ,….in (5) in turn , it is seen in general that

$$7^i (MT_{(i+1)n} - M_{(i+1)n} - 1) = (MT_n - M_n - 1)^{i+1}, i = 1,2,3,...$$

Therefore ,

$$\sum_{i=1}^{k} 7^i (MT_{(i+1)n} - M_{(i+1)n} - 1) = \sum_{i=1}^{k} (MT_n - M_n - 1)^{i+1} \quad (8)$$

Observe that

$$\sum_{i=1}^{k} (MT_n - M_n - 1)^{i+!} = \frac{(MT_n - M_n - 1)^2 ((MT_n - M_n - 1)^k - 1)}{MT_n - M_n - 2} \quad (9)$$

Substituting (9) in (8) ,the required result is obtained.

Property 10: $48\sum_{i=1}^{k} (MT_i - M_i - 1)^2 = 7^4 MS_{2k-1}$

Proof

Consider (7) of Property 9. We have

$$\sum_{n=1}^{k} (MT_n - M_n - 1)^2 = 7\sum_{n=1}^{k} (MT_{2n} - M_{2n} - 1)$$

$$= 7\sum_{n=1}^{k} (2^{2n+1} + 7^{2n+1} - 2^{2n+1} + 1 - 1)$$

$$= 7\sum_{n=1}^{k} 7^{2n+1}$$

$$= 7^2 \sum_{n=1}^{k} 7^{2n}$$

$$= 7^4 \frac{(7^{2k} - 1)}{(7^2 - 1)}$$

$$= 7^4 \frac{MS_{2k-1}}{48}$$

Hence the property.

Property 11: $(MT_n)^2 = MT_{2n+1} + 2*(M_n + 1)(MS_n + 1)$

Proof

$$L.H.S. = (2^{n+1} + 7^{n+1})^2 = (2^{2n+2} + 7^{2n+2}) + 2*2^{n+1}*7^{n+1}$$
$$= MT_{2n+1} + 2*(M_n + 1)*(MS_n + 1) = R.H.S.$$

Property 12: $(MT_n)^3 = MT_{3n+2} + 3*(MT_n)*(M_n + 1)*(MS_n + 1)$

Proof

$$L.H.S. = (2^{n+1} + 7^{n+1})^3 = (2^{3n+3} + 7^{3n+3}) + 3*2^{n+1}*7^{n+1}*(2^{n+1} + 7^{n+1})$$
$$= MT_{3n+2} + 3*(M_n + 1)*(MS_n + 1)*(MT_n)$$

Property 13: $(MT_n)^3 = 3*(MT_n)*(MT_{2n+1}) - 2*(MT_{3n+2})$

Proof

From **Property 11** and **Property 12 ,**we have

$$3*(MT_n)*[(MT_n)^2 - (MT_{2n+1})] = 2*[(MT_n)^3 - (MT_{3n+2})]$$

Thus ,we have

$$(MT_n)^3 = 3*(MT_n)*(MT_{2n+1}) - 2*(MT_{3n+2})$$

Property 14: $(MT_n)^3 + 18*(MT_{3n+1}) = 28*(MT_{3n}) + 3*(MT_n)*(MT_{2n+1})$

Proof

In (1) ,replacing n by $3n$,we get

$$MT_{3n+2} = 9*MT_{3n+1} - 14*MT_{3n}$$

In view of **Property 13 ,** the required result is obtained.

For simplicity and brevity ,some more properties satisfied by Mag-Thiru i numbers are exhibited below:

Property 15 : $(MT_n - M_n - 1)\ (MT_n - M_n)$ is twice the Triangular number of rank 7^{n+1}

Property 16 : $(MT_n - M_n - 1)\ (MT_n - M_n)\ (MT_n - M_n + 1)$ is six times the Triangular pyramidal number of rank 7^{n+1}

Property 17: $(MT_n - M_n - 1)^2\ (MT_n - M_n)$ is twice the Pentagonal pyramidal number of rank 7^{n+1}

Property 18 : $(MT_n - MS_n)^2 = (M_n)^2 + M_{n+2} + 1$

Conclusion

In this paper , a new sequence of integers named as Mag-Thiru numbers have been introduced along with interesting relations among Mag-Thiru numbers. One may search for other connections among Mag-Thiru numbers.

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