ON FINDING INTEGER SOLUTIONS TO SEXTIC EQUATION WITH FOUR UNKNOWNS $xy(x+y)=8zw^5$

Dr. J. Shanthi [1*], Dr. N. Thiruniraiselvi[2], Dr. M.A.Gopalan[3]

[1] Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

[2]Assistant Professor, Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Samayapuram, Trichy- 621 112, Tamil Nadu, India.

[3]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002,Tamil Nadu, India.

Abstract

This paper deals with the problem of finding non-zero distinct integer solutions to the non-homogeneous sextic equation with four unknowns given by $xy(x+y)=8zw^5$.

Key words: Non-homogeneous Sextic , Sextic with four unknowns, integer solutions.
Mathematical Subject Classification 2010: 11D99.

Introduction

It is well-known that a diophantine equation is an algebraic equation with integer coefficients involving two or more unknowns such that the only solutions focused are integer solutions .No doubt that diophantine equations are rich in variety [1-4] .There is no universal method available to know whether a diophantine equation has a solution or finding all solutions if it exists .For equations with more than three variables and degree at least three, very little is known. It seems that much work has not been done in solving higher degree diophantine equations. While focusing the attention on solving sextic Diophantine equations with variables at least three ,the problems illustrated in [5-24] are observed. This paper focuses on finding integer solutions to the sextic equation with four unknowns $xy(x+y)=8zw^5$.

Method of analysis:

The non-homogeneous Diophantine equation of degree six with four unknowns to be solved in integers is

$$xy(x+y)=8zw^5 \quad (1)$$

The process of determining non-zero distinct integer solutions to (1) are illustrated below:

Illustration 1:

Introduction of the transformations

$$x=u+v, y=u-v, z=u \quad (2)$$

in (1) leads to

$$u^2-v^2=4w^5 \quad (3)$$

which is expressed as the system of double equations as below in Table 1:

Table 1: System of double equations

System	I	II	III	IV	V	VI	VII	VIII
$u+v$	$2w^5$	w^5	w^4	$2w^4$	$4w^4$	$4w^3$	$2w^3$	w^3
$u-v$	2	4	$4w$	$2w$	w	w^2	$2w^2$	$4w^2$

In view of (2) , the corresponding integer solutions to (1) are exhibited below:

Solutions from System 1:

$$x=2k^5, y=2, z=k^5+1, w=k$$

Solutions from System II:

$$x=32k^5, y=4, z=16k^5+2, w=2k$$

Solutions from System III:

$$x=16k^4, y=8k, z=8k^4+4k, w=2k$$

Solutions from System IV:

$$x=2k^4, y=2k, z=k^4+k, w=k$$

Solutions from System V:

$$x=64k^4, y=2k, z=32k^4+k, w=2k$$

Solutions from System VI:

$$x=32k^3, y=4k^2, z=16k^3+2k^2, w=2k$$

Solutions from System VII:

$$x = 2k^3, y = 2k^2, z = k^3 + k^2, w = k$$

Solutions from System VIII:

$$x = 8k^3, y = 16k^2, z = 4k^3 + 8k^2, w = 2k$$

Note 1:

Taking

$$v = w^2$$

in (3) ,we get after some algebra

$$x = 2k^2(k+1)^3, y = 2k^3(k+1)^2, z = (2k+1)k^2(k+1)^2, w = k(k+1)$$

which satisfy (1).

Illustration 2 :

Introduction of the transformations

$$x = 2u + v, y = 2u - v, z = u \tag{4}$$

in (1) leads to

$$4u^2 - v^2 = 2w^5 \tag{5}$$

which is expressed as the system of double equations as below in Table 2:

Table 2: System of double equations

System	I	II	III	IV
$2u+v$	w^4	$2w^4$	$2w^3$	$2w^2$
$2u-v$	$2w$	w	w^2	w^3

Solving each system of double equations in Table 2 and using (4) ,the corresponding integer solutions to (1) thus obtained are exhibited below:

Solutions from System I :

$$x = 16k^4, y = 4k, z = 4k^4 + k, w = 2k$$

Solutions from System II :

$$x = 512k^4, y = 4k, z = 128k^4 + k, w = 4k$$

Solutions from System III :

$$x = 16k^3, y = 4k^2, z = 4k^3 + k^2, w = 2k$$

Solutions from System IV :

$$x = 8k^2, y = 8k^3, z = 2k^3 + 2k^2, w = 2k$$

Illustration 3:

Introduction of the transformations

$$x = 2p, y = 2q, z = k(p+q) \tag{6}$$

in (1) leads to

$$pq = kw^5 \tag{7}$$

The different values of p, q, w satisfying (7) and the corresponding integer solutions to (1) using (6) are presented in Table 3 below :

Table 3 : Integer solutions

p	q	w	$x(=2p)$	$y(=2q)$	$z(=k(p+q))$
$k\alpha^5$	1	α	$2k\alpha^5$	2	$k(k\alpha^5+1)$
$k\alpha^4$	α	α	$2k\alpha^4$	2α	$k(k\alpha^4+\alpha)$
$k\alpha^3$	α^2	α	$2k\alpha^3$	$2\alpha^2$	$k(k\alpha^3+\alpha^2)$
$k\alpha^2$	α^3	α	$2k\alpha^2$	$2\alpha^3$	$k(k\alpha^2+\alpha^3)$
$k\alpha$	α^4	α	$2k\alpha$	$2\alpha^4$	$k(k\alpha+\alpha^4)$
$k\alpha^5$	k^5	$k\alpha$	$2k\alpha^5$	$2k^5$	$k(k\alpha^5+k^5)$
$k\alpha^4$	$k^5\alpha$	$k\alpha$	$2k\alpha^4$	$2k^5\alpha$	$k(k\alpha^4+k^5\alpha)$
$k\alpha^3$	$k^5\alpha^2$	$k\alpha$	$2k\alpha^3$	$2k^5\alpha^2$	$k(k\alpha^3+k^5\alpha^2)$
$k\alpha^2$	$k^5\alpha^3$	$k\alpha$	$2k\alpha^2$	$2k^5\alpha^3$	$k(k\alpha^2+k^5\alpha^3)$
$k\alpha$	$k^5\alpha^4$	$k\alpha$	$2k\alpha$	$2k^5\alpha^4$	$k(k\alpha+k^5\alpha^4)$

However, there are other choices of integer solutions to (1) that are exhibited as follows:

Choice 1:

$$x=(8k+1)^5-1, y=1, z=8^4*k^5+5*8^3*k^4+640k^3+80k^2+5k, w=8k+1$$

Choice 2:

$$x=32k^5-4, y=4, z=16k^5-2, w=2k$$

Choice 3:

$$x=16k^4-4k, y=4k, z=4k^4-k, w=2k$$

Choice 4:

$$x=16k^4-8k, y=8k, z=8k^4-4k, w=2k$$

Choice 5:

$$x=8k^3-8k^2, y=8k^2, z=2k^3-2k^2, w=2k$$

Conclusion:

In this paper ,an attempt has been made to determine the non-zero distinct integer solutions to the non-homogeneous sextic diophantine equation with four unknowns given in the title through employing transformations. The researchers in this area may search for other choices of transformations to obtain integer solutions to the sextic diophantine equation with four unknowns under consideration.

References:

[1] L.E.Dickson., History of theory of numbers, vol.11, Chelsea publishing company, Newyork (1952).

[2] L.J.Mordell., Diophantine equations, Academic press, London (1969).

[3] R.D.Carmichael , The theory of numbers and Diophantine analysis, Dover publications, Newyork (1959)

[4] S.G.Telang , Number Theory, Tata MC Graw Hill Publishing Company, New Delhi (1996)

[5] M.A.Gopalan, Manju Somanath and N.Vanitha., Parametric Solutions of $x^2-y^6=z^2$, ActaCiencia Indica XXXIII, vol.3, 1083-1085, (2007)

[6] M.A.Gopalan , G.Sangeetha , On the Sextic Equations with three unknowns $x^2-xy+y^2=(k^2+3)^n z^6$ Impact J.Sci.tech, Vol.4, No: 4, 89-93, (2010)

[7] M.A.Gopalan ,A.Vijayashankar, Integral solutions of the Sextic Equation $x^4+y^4+z^4=2w^6$, Indian journal of Mathematics and Mathematical Sciences, Vol.6, No:2, 241-245, (2010)

[8] Gopalan, M.A., Vidhyalakshmi, S., Vijayashankar, A., Integral Solutions of Non-Homogeneous Sextic equation $xy + z^2 = w^6$, Impact J.Sci..tech, Vol.6, No: 1, 47-52, (2012)

[9] M.A.Gopalan.,S. Vidyalakshmi , K.Lakshmi , Integral Solutions of Sextic Equation with Five unknowns $x^3 + y^3 = z^3 + w^3 + 3(x - y)t^5$, IJERST, 1(10), 562-564, (2012)

[10] M.A.Gopalan, S.Vidhyalakshmi., K.Lakshmi, On the Non-Homogeneous Sextic Equation $x^4 + 2(x^2 + w)x^2y^2 + y^4 = z^4$, IJAMA, 4(2), 171-173, Dec (2012)

[11] M.A.Gopalan , S.Vidhyalakshmi., A.Kavitha., Observations on the Homogeneous Sextic Equation with four unknowns $x^3 + y^3 = 2(k^2 + 3)z^5w$, International Journal of Innovative Research in Science, Engineering and Technology, Vol.2, Issue: 5, 1301-1307, (2013)

[12] M.A.Gopalan, , G.Sumathi, S.Vidhyalakshmi , Integral Solutions of Non- homogeneous Sextic Equation with four unknowns $x^4 + y^4 + 16z^4 = 32w^6$, Antarctica J.Math, 10(6), 623-629, (2013)

[13] M.A.Gopalan ,G. Sumathi.,S. Vidhyalakshmi, Integral Solutions of Sextic Non-Homogeneous Equation with Five unknowns $(x^3 + y^3) = z^3 + w^3 + 6(x + y)t^5$, International Journal of Engineering Research, Vol.1, Issue.2, 146-150, (2013)

[14] M.A. Gopalan, G. Sumathi and S. Vidhyalakshmi, “Integral Solutions of $x^6 - y^6 = 4z\left(x^4 + y^4 + 4\left(w^2 + 2\right)^2\right)$ ” interms of Generalised Fibonacci and Lucas Sequences, Diophantus J. Math., 2(2), 71-75, (2013).

[15] M.A. Gopalan, S. Vidhyalakshmi and K. Lakshmi, “Integral Solutions of the Sextic equation with five unknowns $x^6 - 6w^2(xy + z) + y^6 = 2\left(y^2 + w\right)T^4$” International Journal of Scientific and research Publications, Vol.4, issue.7, (July-2014).

[16] M.A. Gopalan, S. Vidhyalakshmi and A. Kavitha, “Integral Solutions of the Sextic equation with three unknowns $(4k - 1)\left(x^2 + y^2\right) - (4k - 2)xy = 4(4k - 1)z^6$”, International Journal of Innovation Sciences and Research, Vol.4, No:7, 323-328, (July-2015).

[17] M.A.Gopalan, S.Aarthy Thangam., A.Kavitha, ., “On Non-homogeneous Sextic equation with five unknowns $2(x - y)\left(x^3 + y^3\right) = 28\left(z^2 - w^2\right)T^4$ ” , Jamal Academic Research Journal (JARJ) , special Issue, 291-295, (ICOMAC- 2015).

[18] K. Meena., S.Vidhyalakshmi,S. Aarthy Thangam, “On Non-homogeneous Sextic equation with five unknowns $(x+y)(x^3-y^3)=26(z^2-w^2)T^4$ ”, Bulletin of Mathematics and Statistics Research, Vol.5, Issue.2, 45-50, (2017).

[19] S.Vidhyalakshmi .,M.A. Gopalan .,S. Aarthy Thangam , “On Non-homogeneous Sextic equation with five unknowns $2(x+y)(x^3-y^3)=39(z^2-w^2)T^4$ ” , Asian Journal of Applied Science and Technology (AJAST), Volume 1, Issue 6, 45-47, (July 2017).

[20] S.Vidhyalakshmi ,S. Aarthy Thangam ,G. Dhanalakshmi ., “On sextic equation with five unknowns $2(x^3+y^3)(x-y)=84(z^2-w^2)P^4$ ”, IJSRP, 7(8), 22-30, (August 2017).

[21] N.Thiruniraiselvi, M.A.Gopalan,Observations on the Sextic Equation with three unknowns $3(x^2+y^2)-2xy=972z^6$, International Journal Of Mathematics, Statistics And Operations Research ,Volume 1; Number 2;,Pp. 93-98 ,2021.

[22] J.Shanthi , S.Vidhyalakshmi , M.A.Gopalan , On Finding Integer Solutions to Sextic Equation with Three Unknowns $x^2+y^2=8z^6$,The Ciencia and Engenharia-Science and Engineering Journal ,11(1) ,350-355 ,2023.

[23] J.Shanthi ,M.A.Gopalan ,On Finding Integer Solutions to Sextic Equation with Three Unknowns $x^2+y^2=64z^6$,IJSET ,11(4) ,2023.

[24] J.Shanthi ,M.A.Gopalan ,On Non-homogeneous Sextic Equation with Three Unknowns $y^2+3x^2=16z^6$,IJRPR ,4(7) ,1984-1988 ,2023.