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UNIFIED FRAMEWORK FOR CURVE MODELING BY FAMILY OF QUATERNARY SCHEMES AND ITS ANALYSIS

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Abstract

Computational mathematics has a part that controls the techniques for making smooth curves and surfaces and for their effective mathematical representation is known as Computer Aided Geometric Design (CAGD). The Laurent polynomial is used in this research work to create a formula for building a family of quaternary subdivision schemes. There has also been introduced a family of subdivision schemes with various values of parameter m, such as $m = 0,1,2,3,4$, and so forth. The suggested family of subdivision schemes was also examined. The family of schemes we offer has $(m+1)$ continuity. We provide a family of schemes with dual parametrization and $(m+1)$ degree of generation. Additionally shown is the suggested schemes' visual performance.

Keywords: Quaternary; subdivision schemes; Laurent polynomial.

AMS Subject Classifications: 65D17, 65D15, 65D10, 65C20.

1 Introduction

Computational mathematics has a part that controls the techniques for making smooth curves and surfaces and for their effective mathematical representation is known as Computer Aided Geometric Design (CAGD). A subdivision plan is created using an iterative technique that uses a restricted set of points to create surfaces and curves. Because they have so many different types of applications, subdivision schemes have grown to be a significant component of computer graphics.

In 2009, Mustafa and Khan [5] recommended the four-point C^3-quaternary approximating scheme based on single shape parameter. Its support is smaller but approximation order and smoothness are greater. Hashmi and Mustafa [10] proposed a method of estimating the error of quaternary subdivision schemes according to the sequence of the initial control points.

For any integers $m > 1$, Siddiqi and Younis [11] presented a process for creating m-point quaternary approximation systems. This technique was developed utilizing the basis function of the B-spline and the Cox-de Boor formula.

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In 2013, new 5-point binary relaxation subdivision scheme depending on the classical 4-point interpolating scheme is presented by Cao and Tan [7]. To check the uniform convergence and C^k continuity of this scheme, polynomial production method is used. The limiting curves will be fractals when w take some precise values. The uniform B-splines subdivision schemes sign is also present in the Dubuc-Deslauriers [6] schemes symbol. The Lane and Riesenfeld algorithm serve as the foundation for the family of subdivision schemes that Hormann and Sabin [8] proposed.

Peng et al. [9] introduced a ternary four-point rational interpolating subdivision scheme and examined the essential and adequate conditions for continuity. They also discussed the error bound evaluation technique for the proposed schemes. The $(2s - 1)$- points for any integer $s \geq 2$ non-stationary binary subdivision schemes are introduced by Ghaffar et al. [1].

1.1 Contribution and findings

We have developed a Laurent polynomial-based formula for a family of quaternary subdivision schemes. We create subdivision schemes in this study using Laurent polynomials. A discussion has taken place on the subdivision scheme analysis. Also, the following findings and contribution from us are:

A novel set of subdivision schemes for curve design with quaternary structure and high degree of continuity and reproduction.

Various values of m can yield the maximum continuity, degree of generation, and H¨ogelder regularity.

At various values of m, the dual subdivision scheme and the primal scheme can be achieved.

There are six sections in this study. The formula and masks for a family of quaternary subdivision schemes were created in Section 2. We shall discuss the many characteristics of subdivision schemes in section 3. In section 4, *Holder¨* regularity is covered. We compare and discuss the uses of the suggested family of subdivision plans in section 5. Section 6 contains conclusions.

2 The new family of $(m + 2)$-point quaternary schemes

This section will provide a general formula based on the Laurent polynomial for building a family of quaternary subdivision schemes.

$$F_m(z) = \frac{2^m (1+z)^{m-2}}{\left(4z^3\right)^{2m}} \left\{ \alpha_0^{m+2}\alpha_1 + \alpha_0^{m+2}\alpha_2 z + \alpha_0^{m+2}\alpha_2 z^2 + \alpha_0^{m+2}\alpha_1 z^3 \right\}. \tag{2.1}$$

Where $\alpha_0^{m+2} = (1+z+z^2+z^3)^{m+2}$, $\alpha_1 = 1-w$ *and* $\alpha_2 = w - \frac{1}{2}$. We obtain the Laurent polynomial of the family of quaternary subdivision schemes by substituting various values of m.

Case-1: After substituting the $m = 1$ in (2.1), we get the Laurent polynomial

$$F_1(z) = \frac{1}{8z^6(1+z)} \left\{ \alpha_0^3\alpha_1 + \alpha_0^3\alpha_2 z + \alpha_0^3\alpha_2 z^2 + \alpha_0^3\alpha_1 z^3 \right\}. \tag{2.2}$$

Where $\alpha_0^3 = (1+z+z^2+z^3)^3,\ \alpha_1 = 1-w\ \ and\ \ \alpha_2 = w-\frac{1}{2}$. The simplified form of (2.2) is

$$\begin{aligned}F_1(z) &= \left(\frac{1}{8}-\frac{1}{8}w\right)z^5+\left(\frac{3}{16}-\frac{1}{8}w\right)z^4+\left(\frac{5}{16}-\frac{1}{8}w\right)z^3+\left(\frac{1}{2}-\frac{1}{8}w\right)z^2\\&+\left(\frac{3}{8}+\frac{1}{4}w\right)z+\left(\frac{1}{2}+\frac{1}{4}w\right)+\left(\frac{1}{2}+\frac{1}{4}w\right)z^{-1}+\left(\frac{3}{8}+\frac{1}{4}w\right)z^{-2}\\&+\left(\frac{1}{2}-\frac{1}{8}w\right)z^{-3}+\left(\frac{5}{16}-\frac{1}{8}w\right)z^{-4}+\left(\frac{3}{16}-\frac{1}{8}w\right)z^{-5}+\left(\frac{1}{8}-\frac{1}{8}w\right)z^{-6}.\end{aligned} \tag{2.3}$$

The 3-point quaternary scheme corresponding to the Laurent polynomial (2.3) is

$$\begin{aligned}\lambda_{4i}^{k+1} &= \left(\frac{1}{2}-\frac{1}{8}w\right)\lambda_{i-1}^k+\left(\frac{3}{8}+\frac{1}{4}w\right)\lambda_i^k+\left(\frac{1}{8}-\frac{1}{8}w\right)\lambda_{i+1}^k,\\\lambda_{4i+1}^{k+1} &= \left(\frac{5}{16}-\frac{1}{8}w\right)\lambda_{i-1}^k+\left(\frac{1}{2}+\frac{1}{4}w\right)\lambda_i^k+\left(\frac{3}{16}-\frac{1}{8}w\right)\lambda_{i+1}^k,\\\lambda_{4i+2}^{k+1} &= \left(\frac{3}{16}-\frac{1}{8}w\right)\lambda_{i-1}^k+\left(\frac{1}{2}+\frac{1}{4}w\right)\lambda_i^k+\left(\frac{5}{16}-\frac{1}{8}w\right)\lambda_{i+1}^k,\\\lambda_{4i+3}^{k+1} &= \left(\frac{1}{8}-\frac{1}{8}w\right)\lambda_{i-1}^k+\left(\frac{3}{8}+\frac{1}{4}w\right)\lambda_i^k+\left(\frac{1}{2}-\frac{1}{8}w\right)\lambda_{i+1}^k.\end{aligned} \tag{2.4}$$

Case-2: After substituting the $m = 2$ in (2.1), we get the Laurent polynomial

$$F_2(z) = \frac{1}{64z^{12}}\left\{\alpha_0^4\alpha_1+\alpha_0^4\alpha_2 z+\alpha_0^4\alpha_2 z^2+\alpha_0^4\alpha_1 z^3\right\}. \tag{2.5}$$

Where $\alpha_0^4 = (1+z+z^2+z^3)^4,\ \alpha_1 = 1-w\ \ and\ \ \alpha_2 = w-\frac{1}{2}$. The simplified form of (2.5) is

$$\begin{aligned}F_2(z) &= \left(\frac{1}{64}-\frac{1}{64}w\right)z^3+\left(\frac{7}{128}-\frac{3}{64}w\right)z^2+\left(\frac{15}{128}-\frac{5}{64}w\right)z+\left(\frac{7}{32}-\frac{7}{64}w\right)+\left(\frac{5}{16}-\frac{5}{64}w\right)z^{-1}\\&+\left(\frac{49}{128}+\frac{1}{64}w\right)z^{-2}+\left(\frac{57}{128}+\frac{7}{64}w\right)z^{-3}+\left(\frac{29}{64}+\frac{13}{64}w\right)z^{-4}+\left(\frac{29}{64}+\frac{13}{64}w\right)z^{-5}\\&+\left(\frac{57}{128}+\frac{7}{64}w\right)z^{-6}+\left(\frac{49}{128}+\frac{1}{64}w\right)z^{-7}+\left(\frac{5}{16}-\frac{5}{64}w\right)z^{-8}+\left(\frac{7}{32}-\frac{7}{64}w\right)z^{-9}\\&+\left(\frac{15}{128}-\frac{5}{64}w\right)z^{-10}+\left(\frac{7}{128}-\frac{3}{64}w\right)z^{-11}+\left(\frac{1}{64}-\frac{1}{64}w\right)z^{-12}\end{aligned} \tag{2.6}$$

The 4-point quaternary scheme corresponding to the Laurent polynomial (2.6) is

$$\begin{aligned}
\lambda_{4i}^{k+1} &= \left(\frac{7}{32}-\frac{7}{64}w\right)\lambda_{i-1}^{k} + \left(\frac{29}{64}+\frac{13}{64}w\right)\lambda_{i}^{k} + \left(\frac{5}{16}-\frac{5}{64}w\right)\lambda_{i+1}^{k} + \left(\frac{1}{64}-\frac{1}{64}w\right)\lambda_{i+2}^{k},\\
\lambda_{4i+1}^{k+1} &= \left(\frac{15}{128}-\frac{5}{64}w\right)\lambda_{i-1}^{k} + \left(\frac{57}{128}+\frac{7}{64}w\right)\lambda_{i}^{k} + \left(\frac{49}{128}+\frac{1}{64}w\right)\lambda_{i+1}^{k} + \left(\frac{7}{128}-\frac{3}{64}w\right)\lambda_{i+2}^{k},\\
\lambda_{4i+2}^{k+1} &= \left(\frac{7}{128}-\frac{3}{64}w\right)\lambda_{i-1}^{k} + \left(\frac{49}{128}+\frac{1}{64}w\right)\lambda_{i}^{k} + \left(\frac{57}{128}+\frac{7}{64}w\right)\lambda_{i+1}^{k} + \left(\frac{15}{128}-\frac{5}{64}w\right)\lambda_{i+2}^{k},\\
\lambda_{4i+3}^{k+1} &= \left(\frac{1}{64}-\frac{1}{64}w\right)\lambda_{i-1}^{k} + \left(\frac{5}{16}-\frac{5}{64}w\right)\lambda_{i}^{k} + \left(\frac{29}{64}+\frac{13}{64}w\right)\lambda_{i+1}^{k} + \left(\frac{7}{32}-\frac{7}{64}w\right)\lambda_{i+2}^{k}.
\end{aligned} \tag{2.7}$$

Case-3: After substituting the m = 3 in (2.1), we get the Laurent polynomial

$$F_3(z) = \frac{(1+z)}{512z^{18}}\left\{\alpha_0^5\alpha_1 + \alpha_0^5\alpha_2 z + \alpha_0^5\alpha_2 z^2 + \alpha_0^5\alpha_1 z^3\right\}. \tag{2.8}$$

Where $\alpha_0^5 = (1+z+z^2+z^3)^5,\ \alpha_1 = 1-w\ \ and\ \ \alpha_2 = w-\frac{1}{2}.$ The simplified form of (3.2) is

$$\begin{aligned}
F_3(z) = &\left(\frac{1}{512}-\frac{1}{512}w\right)z + \left(\frac{11}{1024}-\frac{5}{512}w\right) + \left(\frac{33}{1024}-\frac{13}{512}w\right)z^{-1} + \left(\frac{19}{256}-\frac{25}{512}w\right)z^{-2}\\
&+\left(\frac{71}{512}-\frac{9}{128}w\right)z^{-3} + \left(\frac{111}{512}-\frac{9}{128}w\right)z^{-4} + \left(\frac{153}{512}-\frac{5}{128}w\right)z^{-5} + \left(\frac{189}{512}+\frac{3}{128}w\right)z^{-6}\\
&+\left(\frac{213}{512}+\frac{25}{256}w\right)z^{-7} + \left(\frac{113}{256}+\frac{37}{256}w\right)z^{-8} + \left(\frac{113}{256}+\frac{37}{256}w\right)z^{-9} + \left(\frac{213}{512}+\frac{25}{256}w\right)z^{-10}\\
&+\left(\frac{189}{512}+\frac{3}{128}w\right)z^{-11} + \left(\frac{153}{512}-\frac{5}{128}w\right)z^{-12} + \left(\frac{111}{512}-\frac{9}{128}w\right)z^{-13} + \left(\frac{71}{512}-\frac{9}{128}w\right)z^{-14}\\
&+\left(\frac{19}{256}-\frac{25}{512}w\right)z^{-15} + \left(\frac{33}{1024}-\frac{13}{512}w\right)z^{-16} + \left(\frac{11}{1024}-\frac{5}{512}w\right)z^{-17} + \left(\frac{1}{512}-\frac{1}{512}w\right)z^{-18}.
\end{aligned} \tag{2.9}$$

The 5-point quaternary scheme corresponding to the Laurent polynomial (2.9) is

$$\begin{aligned}
\lambda_{4i}^{k+1} &= \left(\frac{19}{256}-\frac{25}{512}w\right)\lambda_{i-1}^{k} + \left(\frac{189}{512}+\frac{3}{128}w\right)\lambda_{i}^{k} + \left(\frac{213}{512}+\frac{25}{256}w\right)\lambda_{i+1}^{k} + \left(\frac{71}{512}-\frac{9}{128}w\right)\lambda_{i+2}^{k} + \left(\frac{1}{512}-\frac{1}{512}w\right)\lambda_{i+3}^{k},\\
\lambda_{4i+1}^{k+1} &= \left(\frac{33}{1024}-\frac{13}{512}w\right)\lambda_{i-1}^{k} + \left(\frac{-5}{128}+\frac{153}{512}w\right)\lambda_{i}^{k} + \left(\frac{113}{256}+\frac{37}{256}w\right)\lambda_{i+1}^{k} + \left(\frac{111}{512}-\frac{9}{128}w\right)\lambda_{i+2}^{k} + \left(\frac{11}{1024}-\frac{5}{512}w\right)\lambda_{i+3}^{k},\\
\lambda_{4i+2}^{k+1} &= \left(\frac{11}{1024}-\frac{5}{512}w\right)\lambda_{i-1}^{k} + \left(\frac{111}{512}-\frac{9}{128}w\right)\lambda_{i}^{k} + \left(\frac{113}{256}+\frac{37}{256}w\right)\lambda_{i+1}^{k} + \left(\frac{-5}{128}+\frac{153}{512}w\right)\lambda_{i+2}^{k} + \left(\frac{33}{1024}-\frac{13}{512}w\right)\lambda_{i+3}^{k},\\
\lambda_{4i+3}^{k+1} &= \left(\frac{1}{512}-\frac{1}{512}w\right)\lambda_{i-1}^{k} + \left(\frac{71}{512}-\frac{9}{128}w\right)\lambda_{i}^{k} + \left(\frac{213}{512}+\frac{25}{256}w\right)\lambda_{i+1}^{k} + \left(\frac{189}{512}+\frac{3}{128}w\right)\lambda_{i+2}^{k} + \left(\frac{19}{256}-\frac{25}{512}w\right)\lambda_{i+3}^{k}.
\end{aligned} \tag{2.10}$$

Case-4: After substituting the m = 4 in (2.1), we get the Laurent polynomial

$$F_m(z) = \frac{2^4(1+z)^2}{\left(4z^3\right)^8}\left\{\alpha_0^6\alpha_1 + \alpha_0^6\alpha_2 z + \alpha_0^6\alpha_2 z^2 + \alpha_0^6\alpha_1 z^3\right\}. \tag{2.11}$$

Where $\alpha_0^6 = (1+z+z^2+z^3)^6,\ \ \alpha_1 = 1-w\ \ \ and\ \ \ \alpha_2 = w-\frac{1}{2}.$ The simplified form of (2.11) is

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$$\begin{aligned}F_4(z)=&\left(\frac{1}{4096}-\frac{1}{4096}w\right)z^{-1}+\left(\frac{15}{8192}-\frac{7}{4096}w\right)z^{-2}+\left(\frac{59}{8192}-\frac{25}{4096}w\right)z^{-3}+\left(\frac{21}{1024}-\frac{63}{4096}w\right)z^{-4}\\&+\left(\frac{3}{64}-\frac{123}{4096}w\right)z^{-5}+\left(\frac{735}{8192}-\frac{189}{4096}w\right)z^{-6}+\left(\frac{1219}{8192}-\frac{227}{4096}w\right)z^{-7}+\left(\frac{897}{4096}-\frac{197}{4096}w\right)z^{-8}\\&+\left(\frac{595}{2048}-\frac{37}{2048}w\right)z^{-9}+\left(\frac{1447}{4096}+\frac{61}{2048}w\right)z^{-10}+\left(\frac{1635}{4096}+\frac{163}{2048}w\right)z^{-11}+\left(\frac{433}{1024}+\frac{229}{2048}w\right)z^{-12}\\&+\left(\frac{433}{1024}+\frac{229}{2048}w\right)z^{-13}+\left(\frac{1635}{4096}+\frac{163}{2048}w\right)z^{-14}+\left(\frac{1447}{4096}+\frac{61}{2048}w\right)z^{-15}+\left(\frac{595}{2048}-\frac{37}{2048}w\right)z^{-16}\\&+\left(\frac{897}{4096}-\frac{197}{4096}w\right)z^{-17}+\left(\frac{1219}{8192}-\frac{227}{4096}w\right)z^{-18}+\left(\frac{735}{8192}-\frac{189}{4096}w\right)z^{-19}+\left(\frac{3}{64}-\frac{123}{4096}w\right)z^{-20}\\&+\left(\frac{21}{1024}-\frac{63}{4096}w\right)z^{-21}+\left(\frac{59}{8192}-\frac{25}{4096}w\right)z^{-22}+\left(\frac{15}{8192}-\frac{7}{4096}w\right)z^{-23}+\left(\frac{1}{4096}-\frac{1}{4096}w\right)z^{-24}.\end{aligned} \tag{2.12}$$

The 6-point quaternary scheme is

$$\begin{aligned}\lambda_{4i}^{k+1}=&\left(\frac{21}{1024}-\frac{63}{4096}w\right)\lambda_{i-1}^{k}+\left(\frac{897}{4096}-\frac{197}{4096}w\right)\lambda_{i}^{k}+\left(\frac{433}{1024}+\frac{229}{2048}w\right)\lambda_{i+1}^{k}+\left(\frac{595}{2048}-\frac{37}{2048}w\right)\lambda_{i+2}^{k}\\&+\left(\frac{3}{64}-\frac{123}{4096}w\right)\lambda_{i+3}^{k}+\left(\frac{1}{4096}-\frac{1}{4096}w\right)\lambda_{i+4}^{k},\\\lambda_{4i+1}^{k+1}=&\left(\frac{59}{8192}-\frac{25}{4096}w\right)\lambda_{i-1}^{k}+\left(\frac{1219}{8192}-\frac{227}{4096}w\right)\lambda_{i}^{k}+\left(\frac{1635}{4096}+\frac{163}{2048}w\right)\lambda_{i+1}^{k}+\left(\frac{1447}{4096}+\frac{61}{2048}w\right)\lambda_{i+2}^{k}\\&+\left(\frac{735}{8192}-\frac{189}{4096}w\right)\lambda_{i+3}^{k}+\left(\frac{15}{8192}-\frac{7}{4096}w\right)\lambda_{i+4}^{k},\\\lambda_{4i+2}^{k+1}=&\left(\frac{15}{8192}-\frac{7}{4096}w\right)\lambda_{i-1}^{k}+\left(\frac{735}{8192}-\frac{189}{4096}w\right)\lambda_{i}^{k}+\left(\frac{1447}{4096}+\frac{61}{2048}w\right)\lambda_{i+1}^{k}+\left(\frac{1635}{4096}+\frac{163}{2048}w\right)\lambda_{i+2}^{k}\\&+\left(\frac{1219}{8192}-\frac{227}{4096}w\right)\lambda_{i+3}^{k}+\left(\frac{59}{8192}-\frac{25}{4096}w\right)\lambda_{i+4}^{k},\\\lambda_{4i+3}^{k+1}=&\left(\frac{1}{4096}-\frac{1}{4096}w\right)\lambda_{i-1}^{k}+\left(\frac{3}{64}-\frac{123}{4096}w\right))\lambda_{i}^{k}+\left(\frac{595}{2048}-\frac{37}{2048}w\right)\lambda_{i+1}^{k}+\left(\frac{433}{1024}+\frac{229}{2048}w\right)\lambda_{i+2}^{k}\\&+\left(\frac{897}{4096}-\frac{197}{4096}w\right)\lambda_{i+3}^{k}+\left(\frac{21}{1024}-\frac{63}{4096}w\right)\lambda_{i+4}^{k}.\end{aligned} \tag{2.13}$$

The family of $(m + 2)$-point quaternary approximation subdivision scheme with one parameter is obtained in a similar manner for various values of m. Once several values of m are entered in (2.1), Table 1 displays the complexity and mask of the proposed family of quaternary subdivision schemes.

Table 1: *Shows the mask of a family of quaternary subdivision schemes corresponding to different values of m, here p shows complexity of the schemes (i.e. 3-, 4-, 5-, 6-, …-point schemes).*

m	p	Mask
1	3	$F_1=\Big[(\frac{1}{8}-\frac{1}{8}w),(\frac{3}{16}-\frac{1}{8}w),(\frac{5}{16}-\frac{1}{8}w),(\frac{1}{2}-\frac{1}{8}w),(\frac{3}{8}+\frac{1}{4}w),(\frac{1}{2}+\frac{1}{4}w),$ $(\frac{1}{2}+\frac{1}{4}w),(\frac{3}{8}+\frac{1}{4}w),(\frac{1}{2}-\frac{1}{8}w),(\frac{5}{16}-\frac{1}{8}w),(\frac{3}{16}-\frac{1}{8}w),(\frac{1}{8}-\frac{1}{8}w)\Big]$
2	4	$F_2=\Big[(\frac{1}{64}-\frac{1}{64}w),(\frac{7}{128}-\frac{3}{64}w),(\frac{15}{128}-\frac{5}{64}w),(\frac{7}{32}-\frac{7}{64}w),(\frac{5}{16}-\frac{5}{64}w),$ $(\frac{49}{128}+\frac{1}{64}w),(\frac{57}{128}+\frac{7}{64}w),(\frac{29}{64}+\frac{13}{64}w),(\frac{29}{64}+\frac{13}{64}w),(\frac{57}{128}+\frac{7}{64}w),$ $(\frac{49}{128}+\frac{1}{64}w),(\frac{5}{16}-\frac{5}{64}w),(\frac{7}{32}-\frac{7}{64}w),(\frac{15}{128}-\frac{5}{64}w),(\frac{7}{128}-\frac{3}{64}w),$ $(\frac{1}{64}-\frac{1}{64}w)\Big]$
3	5	$F_3=\Big[(\frac{1}{512}-\frac{1}{512}w),(\frac{11}{1024}-\frac{5}{512}w),(\frac{33}{1024}-\frac{13}{512}w),(\frac{19}{256}-\frac{25}{512}w),$ $(\frac{71}{512}-\frac{9}{128}w),(\frac{111}{512}-\frac{9}{128}w),(\frac{-5}{128}+\frac{153}{512}w),(\frac{189}{512}+\frac{3}{128}w),$ $(\frac{213}{512}+\frac{25}{256}w),(\frac{113}{256}+\frac{37}{256}w),(\frac{113}{256}+\frac{37}{256}w),(\frac{213}{512}+\frac{25}{256}w),$ $(\frac{189}{512}+\frac{3}{128}w),(\frac{-5}{128}+\frac{153}{512}w),(\frac{111}{512}-\frac{9}{128}w),(\frac{71}{512}-\frac{9}{128}w),$ $(\frac{19}{256}-\frac{25}{512}w),(\frac{33}{1024}-\frac{13}{512}w),(\frac{11}{1024}-\frac{5}{512}w),(\frac{1}{512}-\frac{1}{512}w)\Big]$

Analysis of the Scheme

An study of significant characteristics of suggested subdivision designs is presented in this section. Calculating the degree of generation, degree of reproduction, and continuity analysis is done using the Laurent polynomial [3]. While the lower and upper bounds on H¨older regularity of the scheme corresponding to $F_1(z)$ are computed using Rioul's approach [12].

Theorem 3.1. *The family of* $(m + 2)$*-point quaternary approximating subdivision schemes* (2.1) *satisfies the necessary and sufficient conditions of quaternary subdivision scheme.*

Proof. After substituting $z = 1$ in (2.1), we have

$$F_m(1)=\frac{2^m(2)^{m-2}}{(4)^{2m}}\left\{\alpha_0^{m+2}\alpha_1+\alpha_0^{m+2}\alpha_2+\alpha_0^{m+2}\alpha_2+\alpha_0^{m+2}\alpha_1\right\}. \tag{3.1}$$

After simplifications, we get $F_m(1) = 4$. Now for second condition i.e $F_m(z) = 0$ where $z=e^{\frac{2p\pi}{4}i}$ *and* $p=1,2$ *and* 3, $z=(\cos\frac{2p\pi}{4}+i\sin\frac{2p\pi}{4})$ *and* $z^n=(\cos\frac{2p\pi}{4}+i\sin\frac{2p\pi}{4})^n$, *when* $n=1,2$ *and* 3.

For $p = 1$, $z^n = i^n$, then the Laurent polynomial (2.5) takes the form

$$F_m\left(e^{\frac{2\pi}{4}i}\right)=\frac{2^m(1+i)^{m-2}}{\left(4z^3\right)^{2m}}\left\{\alpha_0^{m+2}\alpha_1+\alpha_0^{m+2}\alpha_2 i+\alpha_0^{m+2}\alpha_2 i^2+\alpha_0^{m+2}\alpha_1 i^3\right\}.$$

After simplification, we get $F_m\left(e^{\frac{2\pi}{4}i}\right)=0.$ For $p = 2$ then $z^n = (-1)^n$ the Laurent polynomial (2.1) takes the form

$$F_m\left(e^{\frac{4\pi}{4}i}\right)=\frac{2^m(1-1)^{m-2}}{\left(4z^3\right)^{2m}}\left\{\alpha_0^{m+2}\alpha_1-\alpha_0^{m+2}\alpha_2+\alpha_0^{m+2}\alpha_2-\alpha_0^{m+2}\alpha_1\right\}.$$

After simplification, we get $F_m\left(e^{\frac{4\pi}{4}i}\right)=0.$ Similarly, for $p = 3$ then $z^n = (-i)^n$ the Laurent polynomial (2.1) takes the form

$$F_1\left(e^{\frac{6\pi}{4}i}\right)=\frac{2^m(1-i)^{m-2}}{\left(4z^3\right)^{2m}}\left\{\alpha_0^{m+2}\alpha_1-\alpha_0^{m+2}\alpha_2 i+\alpha_0^{m+2}\alpha_2 i^2-\alpha_0^{m+2}\alpha_1 i^3\right\}.$$

Hence, we have $F_m(1) = 4$ and $F_m\left(e^{\frac{2p\pi}{4}i}\right)=0$, for $p=1,2$ *and* $3,$ So the schemes corresponding to $F_m(z)$ satisfies the necessary condition of convergence. For sufficient condition of convergence, if F_m is the scheme corresponding to the Laurent polynomial $F_m(z)$ then we can easily prove that the $\left\|\frac{1}{4}F_m\right\|_\infty<1$, for different cases. So the scheme corresponding to $F_m(z)$ satisfies the sufficient condition of convergence.

Which completes the proof. □

Theorem 3.2. *The family of (m + 2)-point quaternary subdivision schemes corresponding to Laurent polynomial (2.1) is C^{m+1} continuous for some w.*

Proof. For C^{m+1} continuity of the schemes corresponding to $F_m(z)$ defined in (2.1), consider the Laurent polynomial

$$F_m^1(z)=\left(\frac{4z^3}{1+z+z^2+z^3}\right)^{m+2}F_m(z),$$

where $F_m(z)$ is defined in (2.1). This implies

$$F_m^1(z)=\left(\frac{4z^3}{1+z+z^2+z^3}\right)^{m+2}\frac{2^m(1+z)^{m-2}}{\left(4z^3\right)^{2m}}\alpha_0^{m+2}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}.$$

Where $\alpha_0^{m+2}=(1+z+z^2+z^3)^{m+2},\ \alpha_1=1-w$ *and* $\alpha_2=w-\frac{1}{2}.$ After simplification, we get

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$$F_m^1(z)=\frac{2^m(1+z)^{m-2}}{\left(4z^3\right)^{m-2}}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}. \tag{3.2}$$

Let S_m be the scheme corresponding to $F_m^1(z)$. The scheme corresponding to $F_m(z)$ is C^{m+1} continuous if $\|\frac{1}{4}S_m\|_\infty<1,$ for this, we have to check that

Case-1: If m = 1 then $F_1^1(z)=8z^3(1+z)^3\left(\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right),$ we have the mask of the scheme $S_1=[|2w-2|,|4w-3|,|2w-2|]$. *Since* $\|\frac{1}{4}S_1\|_\infty=\max\{|2w-2|,|4w-3|,|2w-2|\}$, S_1 = $[|2w-2|,|4w-3|,|2w-2|]$. This norm $\|\frac{1}{4}S_1\|_\infty<1,$ *when* $w\in(\frac{1}{2},1).$

Case-2: If m = 2 then $F_2^1(z)=4\left(\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}\right),$ we have the mask of the scheme $S_2=4\left[|1-w|,\left|w-\frac{1}{2}\right|,\left|w-\frac{1}{2}\right|,|1-w|\right]$. *Since* $\|\frac{1}{4}S_2\|_\infty=\max\left\{|1-w|,\left|w-\frac{1}{2}\right|\right\}$, this norm $\|\frac{1}{4}S_2\|_\infty<1,$ *when* $w\in(0,\frac{3}{2}).$

Case-3: If m = 3 then $F_3^1(z)=2z^{-3}(1+z)\left(\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right)$, we have the mask of the scheme $S_3=\left[|4w-4|,|4w-2|,|1|,|1|\right]$. *Since* $\|\frac{1}{4}S_3\|_\infty=\max\left\{|w-1|,\left|w-\frac{1}{2}\right|\right\}$, this norm $\|\frac{1}{4}S_3\|_\infty<1,$ *when* $w\in(0,\frac{3}{2})$

In general, the norm $\|\frac{1}{4}S_\beta\|_\infty<1$ for different values of m. The schemes corresponding to $F_m(z)$ has C^{m+1} continuity. Which completes the proof. □

Theorem 3.3. *The degree of the polynomial generation of family of* (m+2)*-point quaternary subdivision scheme corresponding to* $F_m(z)$ *is* m + 1.

Proof. From (2.1), we can re-write as

$$F_m(z)=\left(\frac{1+z+z^2+z^3}{4z^3}\right)^{(m+1)+1}b_m(z).$$

Where,

$$b_m(z)=\frac{2^m(1+z)^{m-2}}{\left(4z^3\right)^{m-2}}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}.$$

So the degree of polynomial generation is m + 1. Which completes the proof. □

Table 2: *Shows the continuity of family of quaternary subdivision schemes corresponding to different values of m.*

Cases	Scheme	Continuity	Range of w
$m = 0$	F_0	C_0 C_1	$w \in (0.5, 1.25)$ $w =$ [illegible]
$m = 1$	F_1	C_0 C_1 C_2	$w \in (-1.5, 2.5)$ $w \in (-0.25, 1.75)$ $w \in (0.5, 1)$
$m = 2$	F_2	C_0 C_1 C_2 C_3	$w \in (-7.75, 8)$ $w \in (-3.75, 4.25)$ $w \in (-1.5, 2.5)$ $w \in (0, 1.5)$
$m = 3$	F_3	C_0 C_1 C_2 C_3 C_4	$w \in (-9.615, 10.0767)$ $w \in (-7.875, 8.125)$ $w \in (-3.75, 4.25)$ $w \in (-3.5, 4)$ $w \in (0, 1.5)$
$m = 4$	F_4	C_0 C_1 C_2 C_3 C_4 C_5	$w \in (-13.4324, 14.1613)$ $w \in (-10.8586, 11.4021)$ $w \in (-7.875, 8.125)$ $w \in (-5.166, 5.5)$ $w \in (-3.75, 4)$ $w \in (-0.75, 2.25)$

Theorem 3.4. *The family of* $(m+2)$*-point quaternary subdivision scheme corresponding to* $F_m(z)$ *has* $(m+1)$*th degree of reproduction with respect to the dual parametrization.*

Proof. By taking the derivative of (2.1) with respect to z, we get

$$F_m'(z) = \frac{2^m(1+z)^{m-2}(m-2)\alpha_0^{m+2}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}}{(1+z)(4z^3)^{2m}}$$
$$+\frac{2^m(1+z)^{m-2}(m-2)\alpha_0^{m+1}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}(1+2z+3z^2)}{(4z^3)^{2m}} \tag{3.3}$$
$$+\frac{2^m(1+z)^{m-2}\alpha_0^{m+2}\left\{\alpha_2+2z\alpha_2+3\alpha_1 z^2\right\}}{(4z^3)^{2m}}-\frac{6m2^m(1+z)^{m-2}\alpha_0^{m+2}\left\{\alpha_1+\alpha_2 z+\alpha_2 z\ 2+\alpha_1 z^3\right\}}{(4z^3)^{2m}z}.$$

After taking the higher derivative of (3.3) and substituting $z = 1$, we have

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$$F_m^{''}(1)=\frac{1}{4^{m-2}}2^{(2m-3)}(32m^2-45m+30-8w),$$
$$F_m^{'''}(1)=240-256m^3-539m+408m^2-120w+192mw,$$
$$F_m^{''''}(1)=840-1472m^3-1536m^2w-2407m+1024m^4+3227m^2-576w+1392mw, \quad (3.4)$$
$$F_m^{'''''}(1)=2340-16700m^3-4096m^5+10240m^3w-8640m^2w$$
$$-\frac{18033}{2}m+3840m^4+\frac{26685}{2}m^2-1920w+6920mw.$$

The value of shift parameter $\tau=\frac{F_m^{'}(1)}{4}=-4m+\frac{7}{2}$. Hence by [3] the subdivision scheme (2.1) has dual parametrization. Further, we can easily verify that

$$F_m^k(1)=4\prod_{j=0}^{k-1}(-4m+\frac{7}{2}-j) \quad \text{for} \quad k=0,1,\cdots,m+1.$$

Hence by [3] the family of schemes corresponding to $F_m(z)$ has $(m+1)$ degree of reproduction with respect to the dual parametrization. □

The Table 3 summarized the results of degree of generation, degree of reproduction and range of parameter different cases corresponding to m.

Table 3: *Shows the Degree of generation and Degree of reproduction of family of Quaternary subdivision schemes corresponding to different values of* m.

Cases	Scheme	G_d	R_d	Range of w
$m=0$	F_0	1	Linear	if $w=\frac{7}{8}$
$m=1$	F_1	2	Linear Quadratic	 if $w=\frac{31}{16}$
$m=2$	F_2	3	Linear Quadratic Cubic	 if $w=\frac{37}{16}$ if $w=\frac{37}{16}$
$m=3$	F_3	4	Linear Quadratic Cubic Quartic	 if $w=\frac{43}{16}$ if $w=\frac{43}{16}$ if $w=$ [illegible]
$m=4$	F_4	5	Linear Quadratic Cubic Quartic Quantic	 if $w=\frac{43}{16}$ if $w=\frac{43}{16}$ if $w=$ [illegible] if $w=$ [illegible]

Theorem 3.5. *The H¨older regularity of the family of quaternary subdivision schemes corresponding to* (2.1) *is* $m + 2$.

Proof. The Laurent polynomial of the family of quaternary subdivision schemes $F_m(z)$ can be written as

$$F_m(z) = \left(\frac{1+z+z^2+z^3}{4z^3}\right)^{m+2} b_m(z), \tag{3.5}$$

Where

$$b_m(z) = \frac{2^m(1+z)^{m-2}}{\left(4z^3\right)^{m-2}}\left\{\alpha_1+\alpha_2 z+\alpha_2 z^2+\alpha_1 z^3\right\}. \tag{3.6}$$

From (3.5), we have $k = m+2$ (i.e. number of factors in $F_m(z)$). Let $\beta_0, \beta_1, \cdots, \beta_{m+1}$, (i.e. non-zero coefficients of z in $b_m(z)$), $q = 0,1,2,\cdots,m+1$ (i.e. number of non-zero coefficients of z in $b_m(z)$, start counting from 0). The entries of $(B_0)_{i,j} = \beta_{2+i-4j}, (B_1)_{i,j} = \beta_{3+i-4j}, \cdots, (B_{m+1})_{i,j} = \beta_{m+2+i-4j}$ can be easily find. The Holder regularity $r = k - \log_4\mu$, where μ is the joint spectral radius of the matrices $B_0, B_1, B_2, \cdots, B_{m+1}$, that is, $\mu = \rho(B_0, B_1, B_2, \cdots, B_{m+1})$. For bounds on Holder regularity we calculate $max\{\rho(B_0), \rho(B_1), \cdots, \rho(B_{m+1}),\} \le \mu \le max\{\|(B_0)\|_\infty, \|(B_1)\|_\infty, \cdots, \|(B_{m+1})\|_\infty\}$.

Case-1: For $m = 1$, the lower and upper bounds of Holder regularity are $r \ge 3 - \log_4(\mu)$ and $r \le 3 - \log_4(\nu)$ respectively, where

$$\mu = \begin{cases} |-8+8w| & \text{if} \quad 0.5 < w \le 0.83, \\ |16w-12| & \text{if} \quad 0.83 < w < 1. \end{cases}$$

and

$$\nu = \begin{cases} 8-8w & \text{if} \quad 0.5 < w \le 0.83, \\ 16w-12 & \text{if} \quad 0.83 < w < 1. \end{cases}$$

Case-2: For $m = 2$, the lower and upper bounds of Holder regularity are $r \ge 4 - \log_4(\mu)$ and $r \le 4 - \log_4(\nu)$ respectively, where

$$\mu = \begin{cases} |-4+4w| & \text{if} \quad 0 < w \le 0.75, \\ |4w-2| & \text{if} \quad 0.75 \le w < 1.5. \end{cases}$$

and

$$\nu = \begin{cases} 4-4w & \text{if} \quad 0 < w \le 0.75, \\ 4w-2 & \text{if} \quad 0.75 \le w < 1.5. \end{cases}$$

Case-3: For $m = 3$, the lower and upper bounds of Ho¨lder regularity are $r \ge 5 - \log_4(\mu)$ and $r \le 5 - \log_4(\nu)$ respectively, where

$$\mu = \begin{cases} 2|2-2w| & \text{if} \quad 0 < w \le 0.75, \\ |4w-2| & \text{if} \quad 0.75 \le w < 1.5. \end{cases}$$

and

$$\nu = \begin{cases} 2-2w & \text{if} \quad 0 < w \le 0.5, \\ 1 & \text{if} \quad 0.5 \le w \le 0.75, \\ 4w-2 & \text{if} \quad 0.75 \le w < 1.25. \end{cases}$$

Case-4: For $m = 4$, the lower and upper bounds of Holder regularity are $r \geq 6 - \log_4(\mu)$ and $r \leq 6 - \log_4(\nu)$ respectively, where

$$\mu = \begin{cases} |w - \frac{3}{2}| + |w - 1| & \text{if} \quad -0.75 < w \leq 0.75, \\ |\frac{-1}{2} + 2w| & \text{if} \quad 0.75 \leq w < 2.25. \end{cases}$$

and

$$\nu = \begin{cases} \frac{5}{2} - 2w & \text{if} \quad -0.75 < w \leq 0.75, \\ \frac{-1}{2} + 2w & \text{if} \quad 0.75 \leq w < 2.25. \end{cases}$$

The Holder regularity of the family of schemes corresponding to the symbol $F_m(z)$ is $m + 2$. This completes the proof. □

Theorem 3.6. *The limit stencils providing the evaluations of the basic limit function of the 3-point scheme (2.4) at integers are* $\left\{\frac{1}{15} - \frac{w}{15}, \frac{13}{30} + \frac{w}{15}, \frac{13}{30} + \frac{w}{15}, \frac{1}{15} - \frac{w}{15}\right\}$.

Proof. Put $i = -1$ and $i = 0$ in 3-point subdivision scheme (2.4), we get the subdivision matrix as follows as S_1.

$$\begin{pmatrix} f_{-2}^{k+1} \\ f_{-1}^{k+1} \\ f_{0}^{k+1} \\ f_{1}^{k+1} \end{pmatrix} = S_1 \begin{pmatrix} f_{-2}^{k} \\ f_{-1}^{k} \\ f_{0}^{k} \\ f_{1}^{k} \end{pmatrix},$$

where S_1 is local subdivision matrix

$$S_1 = \begin{bmatrix} \frac{3}{16} - \frac{1}{8}w & \frac{1}{2} + \frac{1}{4}w & \frac{5}{16} - \frac{1}{8}w & 0 \\ \frac{1}{8} - \frac{1}{8}w & \frac{5}{8} + \frac{1}{4}w & \frac{1}{4} - \frac{1}{8}w & 0 \\ 0 & \frac{1}{4} - \frac{1}{8}w & \frac{5}{8} + \frac{1}{4}w & \frac{1}{8} - \frac{1}{8}w \\ 0 & \frac{5}{16} - \frac{1}{8}w & \frac{1}{2} + \frac{1}{4}w & \frac{3}{16} - \frac{1}{8}w \end{bmatrix}.$$

The eigenvalues of the matrix S_1 are, $\left\{1, \frac{1}{16}, \frac{-3}{16} + \frac{w}{4}, \frac{1}{4}\right\}$. The eigenvectors of local subdivision matrix S_1 corresponding to eigenvalues is

$$Q_1 = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & \frac{-3+2w}{13+2w} & -\frac{-3+2w}{2w+1} & \frac{-1}{3} \\ 1 & \frac{-3+2w}{13+2w} & \frac{-3+2w}{2w+1} & \frac{1}{3} \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

The inverse of Q_1 is

$$Q_1^{-1} = \begin{pmatrix} \frac{1}{15}\frac{-1}{6}w & \frac{13}{30} + \frac{1}{15}w & \frac{13}{30} + \frac{1}{15}w & \frac{1}{15} - \frac{1}{15}w \\ \frac{13}{30} + \frac{1}{15}w & -\frac{13}{30} - \frac{1}{15}w & -\frac{13}{30} - \frac{1}{15}w & \frac{13}{30} + \frac{1}{15}w \\ \frac{1}{2}\frac{2w+1}{-7+4w} & \frac{-3}{2}\frac{2w+1}{-7+4w} & \frac{3}{2}\frac{2w+1}{-7+4w} & \frac{-1}{2}\frac{2w+1}{-7+4w} \\ -3\frac{-1+w}{-7+4w} & \frac{3}{2}\frac{2w+1}{-7+4w} & \frac{-3}{2}\frac{2w+1}{-7+4w} & 3\frac{-1+w}{-7+4w} \end{pmatrix}.$$

For the decomposition of matrix S_1, we need D_1. Where D_1 is the scalar matrix in which eigenvalues are arranged diagonally, therefore, now compute $\lim_{k\to\infty} D_1^k$

$$D_1^k = \begin{pmatrix} (1)^k & 0 & 0 & 0 \\ 0 & (\frac{1}{16})^k & 0 & 0 \\ 0 & 0 & (\frac{-3}{16}+\frac{1}{4}w)^k & 0 \\ 0 & 0 & 0 & (\frac{1}{4})^k \end{pmatrix} \text{ and } \lim_{k\to\infty} D_1^k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Since $f^{k+1} = (Q_1 D_1^k Q_1^{-1}) f^0$, we get $f^\infty = Q_1(\lim_{k\to\infty} D_1^k) Q_1^{-1} f^0$. This implies that

$$\begin{pmatrix} f_{-2}^\infty \\ f_{-1}^\infty \\ f_0^\infty \\ f_1^\infty \end{pmatrix} = \begin{pmatrix} \frac{1}{15}-\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{1}{15}-\frac{1}{15}w \\ \frac{1}{15}-\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{1}{15}-\frac{1}{15}w \\ \frac{1}{15}-\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{1}{15}-\frac{1}{15}w \\ \frac{1}{15}-\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{13}{30}+\frac{1}{15}w & \frac{1}{15}-\frac{1}{15}w \end{pmatrix} \begin{pmatrix} f_{-2}^0 \\ f_{-1}^0 \\ f_0^0 \\ f_1^0 \end{pmatrix}.$$

Hence the limit stencils providing the evaluations of the basic limit function of the 3-point scheme (2.4) at integers are $\left\{\frac{1}{15}-\frac{w}{15}, \frac{13}{30}+\frac{w}{15}, \frac{13}{30}+\frac{w}{15}, \frac{1}{15}-\frac{w}{15}\right\}$. □

Table 4: *Shows the limit stencils of a family of quaternary subdivision schemes corresponding to different values of m.*

F_m	Limit stencils
F_0	$\{\frac{-2+2w}{-5+4w}, \frac{-3+2w}{-5+4w}, 0\}$
F_1	$\{\frac{1}{15}-\frac{w}{15}, \frac{13}{30}+\frac{w}{15}, \frac{13}{30}+\frac{1}{15}w, \frac{1}{15}-\frac{1}{15}w\}$
F_2	$\{-\frac{1}{324}\frac{72w^2-211w+139}{2w-129}, \frac{1}{648}\frac{432w^2+7878w-16819}{2w-129}-\frac{1}{1080}\frac{1008w^2+22718w+62507}{2w-129},$ $\frac{1}{3240}\frac{2448w^2+21578w-133373}{2w-129}, -\frac{1}{3240}\frac{864w^2-11556w+11581}{2w-129}\}$
F_3	$\{\frac{37661}{422601300}-\frac{13249}{84520260}w+\frac{818}{11738925}w^2-\frac{8}{3912975}w^3,$ $\frac{38150593}{845202600}+\frac{8186}{11738925}w^2-\frac{14289491}{422601300}w+\frac{8}{782595}w^3,$ $\frac{129666419}{422601300}-\frac{40924}{11738925}w^2-\frac{3958783}{211300650}w-\frac{16}{782595}w^3,$ $\frac{90862787}{211300650}+\frac{65476}{11738925}w^2+\frac{26902627}{211300650}w+\frac{16}{782595}w^3,$ $\frac{43348127}{211300650}-\frac{45014}{11738925}w^2-\frac{26368621}{422601300}w-\frac{8}{782595}w^3,$ $\frac{10800191}{845202600}+\frac{674}{690525}w^2-\frac{5163331}{422601300}w+\frac{8}{3912975}w^3\}$

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3.1 Applications

The visual performance of the suggested family of subdivision designs will be covered in this section. The smooth curves produced by our suggested systems are represented by complete lines, while the control polygons are represented by dot lines. Limit curves for near polygons are shown in Figures 1.

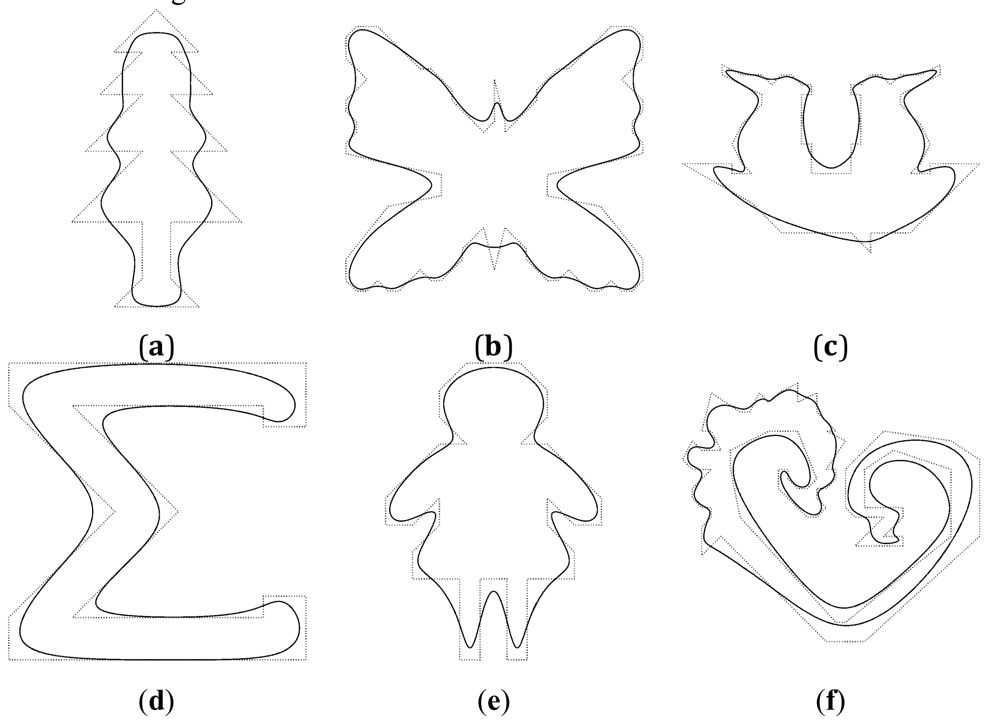

Figure 1: (**a**) − (**c**) *and* (**d**) − (**f**) *present limit curves for close polygons produced by scheme corresponding to* F_1 *and* F_2 *respectively.*

4 Conclusion

This study presents a universal formula that uses the Laurent polynomial to build a variety of quaternary subdivision schemes. There has also been introduced a family of subdivision schemes with various values of parameter m, such as $m = 0,1,2,3,4$, and so forth. The suggested family of subdivision schemes was also examined. The family of schemes we offer has $(m + 1)$ continuity. The dual parametrization and $(m + 1)$ degree of generation of the suggested family of schemes. The suggested scheme's visual performance demonstrates how effective these schemes are at modeling curves.

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