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COMPUTATION OF CHEZY'S RESISTANCE COEFFICIENT IN A PARABOLIC SHAPED-CHANNEL FOR TURBULENT FLOW

Mohammed Lakhdar Nebbar[1*]
[1] Laboratory of Water, Environment and Renewable Energies. Department of hydraulics, Faculty of Technology. University of M'sila, 28000. Algeria
[*] Corresponding: Mohammed Lakhdar Nebbar. E-mail: mohamedlakhdar.nebbar@univ-msila.dz

Abstract:
The Chézy formula describes the average flow velocity in open channel turbulent flow and is widely used in fields related to fluid mechanics and fluid dynamics. Open channels refer to any open conduit, such as rivers, ditches, canals, or partially full pipes. In this context, the calculation of Chezy's resistance coefficient is typically not provided a priori in a design problem, and its value is often selected subjectively from the literature for most open channels and conduits under uniform flow conditions. However, in most practical cases, if these coefficients are not expressed by implicit models, they are generally taken as constant and arbitrary. To this end, and in a rational manner, the dimensioning and design of channels require the expression of the resistance coefficient in an easily and explicitly defined form by adopting numerous flow parameters, namely the roughness of the walls, the aspect ratio, the slope of the channels, and essentially the viscosity of the liquid. To achieve this aim, Chezy's resistance coefficient $\boldsymbol{C}$ is identified using the rough model method (RMM), which provides the discharge under uniform flow conditions appropriate to a parabolic-shaped channel.Some examples are presented showing how to calculate Chezy's resistance coefficient with a minimum practical data.

Keywords: Aspect Ratio; Chezy's Resistance Coefficient; Kinematic Viscosity; Modified Reynolds Number; Parabolic-Shaped Channel; Roughness; Rough Model Method (RMM); Turbulent Flow

1 Introduction

As early as 1769, the French engineer Antoine Chezy [1], ran extensive tests on an earthen canal and the Seine River. He reasoned that the resistance would vary with the wetted perimeter and with the square of velocity, and the force to balance this resistance would vary with the area of cross section and with the slope, and concluded that:

$$V = C\sqrt{R_h I_f} \tag{1}$$

Where V is the mean velocity $(\mathrm{m/s})$, C is the Chezy's resistance coefficient $\left(\mathrm{m^{1/2}/s}\right)$ varies from about 30 $\mathrm{m^{1/2}/s}$ for small rough channels to 90 $\mathrm{m^{1/2}/s}$ for large smooth channels, R_h is the hydraulic radius (m) and I_f is the bed slope of the channel. Eq. (1) is a semi-empirical resistance equation [2], which estimates mean flow velocity in open channel conduits. The approximate character of Chezy's equation for uniform flow in an open channel is widely acknowledged, as can be seen from the numerous formulae for the Chezy coefficient quoted in the literature [3, 4, 5] . The formula (1) had been proven to work also for non-uniform gradually varied flow (GVF) after replacing the bed slope by the energy slope. Both uniform and GVF Chezy forms could be re-written in velocity form instead of slope form Elgamal and Steffler [6].

In the literature [4, 7, 8] , the great deal of hydraulic researchers correlated the Chezy coefficient C with roughness, shape, and slope of various of open channel flow. Among them were Guanguillet and Kutter [9]; Manning [10]; Bazin [11]; Powell [12]. These relationships are well summarized and discussed by Chow [4]).

The Guanguillet and Kutter [9] formula expresses C in terms of the hydraulic radius R_h, the coefficient of roughness n known as Kutter's n and the slope I. In S.I. Units, this formula is:

$$C = \frac{23 + \frac{0.00155}{I_f} + \frac{1}{n}}{1 + \left(23 + \frac{0.00155}{I_f}\right)\frac{n}{\sqrt{R_h}}} \quad (2)$$

The French hydraulician Bazin [11] proposed a formula according to which Chezy's C is considered a function of R_h but not of I_f. Expressed in S.I. Units, this formula is:

$$C = \frac{87}{1 + \frac{k_B}{\sqrt{R_h}}} \quad (3)$$

Where k_B is a coefficient of roughness whose values proposed by Bazin are given by a table as a function of the type of the material forming the channel or the conduit.

After Chezy's formula became generally known, there was a lot of interest in developing formulas to predict the value of C. The most enduring of the resulting empirical prediction formulas is usually attributed, wrongly according to Henderson [7], to Robert Manning. After some modification, Manning's formula Manning [10] became:

$$C = \frac{1}{n} {R_h}^{1/6} \quad (4)$$

Where *n* is the Manning's roughness coefficient. This coefficient is essentially a function of the nature of boundary surface. Note that these relationships (2), (3) and (4) do not contain a kinematic viscosity parameter. They therefore do not apply to the entire domain of turbulent flow.

Powell [12] suggested a logarithmic formula for the roughness of artificial channels. This formula, an implicit function of *C*, in S.I. Units, this formula is:

$$C = -42 log\left[\frac{\varepsilon}{R_h} + \frac{C}{R_e}\right] \quad (5)$$

According to this relationship (5), C depends especially on the Reynolds number R_e(defined as $R_e = 4R_hV/\nu$), in which ν is the kinematic viscosity coefficient. In this relation, there is no term that expresses the influence of the slope I_f on the coefficient C. Its application seems to be suitable for the entire domain of turbulent flow. It is interesting to note that Powell formula contains the absolute roughness ε which is a measurable parameter in practice. The calculation of the coefficient C by Powell's relation requires an iterative process.

Another expression developed for use in pipes is sometimes used in open channels [13,14]. This is the Darcy-Weisbach equation which can be written for open channels [15,16] as

$$C = \left(\frac{8g}{f}\right)^{1/2} \quad \text{or} \quad f = \frac{8g}{C^2} \quad (6)$$

Where f is the Darcy-Weisbach friction factor. For smooth pipes, f is found to be a function of the Reynolds number R_e only. For rough turbulent flows, f is a function of the relative roughness ($\varepsilon/4R_h$) and type of roughness and is independent of the Reynolds number. In the transition regime, both the Reynolds number and relative roughness play important roles. This flow resistance equation (6) is also assuming steady uniform flow. Steady uniform flow is a flow in open channel where the depth of flow does not change, or the flow can be assumed to be constant during the time interval under consideration [17] . In general, uniform flow can occur only in very long, straight, and prismatic channels. Although the definition of uniform flow and the assumptions required to consider equations (2) and (6) are rarely satisfied in practice, the concept of uniform flow is central to the understanding and solution to many practical tasks of open-channel hydraulics [4,15,16]. Recently, Swamee and Rathie [18] have attempted to propose a general relationship for Chezy's coefficient C, applicable in the entire domain of turbulent flow and for all shapes of channels and conduits. This formula is:

$$C = -2.457ln\left(\frac{(\varepsilon/R_h)}{12} + \frac{0.221\nu}{R_h\sqrt{gI_f R_h}}\right) \quad (7)$$

The formula (7) is implicit, requiring also a trial-and-error procedure especially when the linear dimension of the channel or conduit is not given, or when it comes to compute the normal depth of the flow. Apart from its implicit form, this relationship has the advantage of being very complete. All the flow parameters are included in this relationship.

Several other studies have been carried out by researchers to define the Chezy coefficient. However, their uses remain very restricted. They include Pyle et al. [19], Naot and Novak [20], Ead et al. [21] and Giustolisi [22], etc. With this in mind, and with the aim of avoiding these shortcomings, this work contributes to the development of an expression for calculating the Chezy coefficient C to make it more manageable and easier to use. Based on the rough model method (RMM) Achour and Bedjaoui [23] for expressing the discharge in turbulent flow and is valid in all geometric profiles [24-28]. For this reason, this article proposes to establish a general relationship for the explicit calculation of the Chezy coefficient in a parabolic channel, taking into account the required hydraulic parameters, namely, the aspect ratio of the normal water area, the longitudinal bed slope, the absolute roughness of the inner walls of the channel, and the kinematic viscosity of the fluid. This relationship is valid for all cases of turbulent flow in a parabolic-shaped channel (see Fig.1).

We chose the parabolic-shaped channel profile because it is generally used in free-surface flows for evacuating rainwater and draining sewage from cities, as well as for transporting supplies and irrigation water. The parabolic cross-section shape is, in many situations, the best practical shape for an open channel. One of its advantages is the ability to maintain a higher velocity at low discharge, which reduces the tendency to deposit sediment. Parabolic channels provide good water flow conditions, especially in cold areas, and have the advantages of antifreeze and expansion, so they are widely used in agricultural drainage and irrigation, urban water supply and drainage and other projects Zhao et al. [29]. Furthermore, Mironenko et al. [30] and Chahar [31] stated that since river beds, unlined channels, and irrigation furrows all tend to approximate a stable parabolic shape, unlined channels are made more hydraulically stable when they are initially constructed in a parabolic shape. The parabolic-shaped section it has been classified into the group of curved sections.

2 Method

Conducting research in natural field conditions, determining the parabolic parameter in canals with a parabolic cross-section is one of the important issues in the fields of hydraulics and engineering hydrology.

2.1 *Geometric and Hydraulic Properties of Parabolic Section*

The parabolic channel section is displayed in Fig.1, is typically characterized as:

$$Y = KX^2 \quad (8)$$

Where, Y = ordinate; X = abscissa and K is the shape factor of the parabolic channel. The top width T, water area A and wetted perimeter P are given in terms of the maximum channel depth and maximum permissible side slope $1/z$.

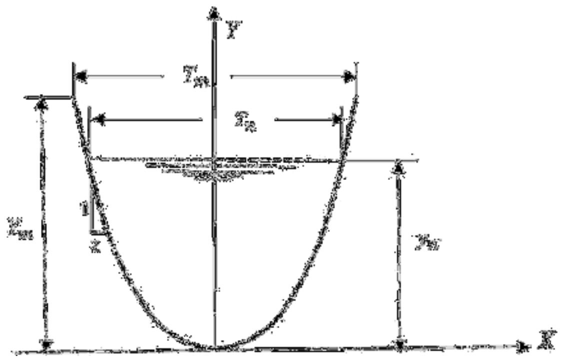

Figure 1: Schematic representation of normal depth in parabolic shaped-channel

According to [32], the channel considered is defined by the linear dimensions T_m, Y_m and y_n. Flowing under a bed slope I_f at a flow rate Q of a liquid with kinematic viscosity ν. The condition of the channel's internal wall is described by the absolute roughness ε.
For $X \geq 0$, where X is the longitudinal coordinate. Three points are particularly considered namely:

The first point $P(T_m/2\,, y_m)$ which is well defined by the geometrical elements T_m and Y_m of the parabolic channel. For this point, Eq. (8) gives: $Y_m = K(T_m/2)^2$ or:

$$K = 4\frac{Y_m}{T_m^2} \tag{9}$$

Inserting Eq. (9) into Eq. (8), results in:

$$Y = \frac{4}{B}X^2 \tag{10}$$

in which:

$$B = \frac{T_m^2}{Y_m} \tag{11}$$

The parameter B is thus a linear dimension and it is well defined by the geometric elements T_m and Y_m.

The second point $N(T_n/2\,, y_n)$ which is connected to the uniform flow characterized by the top width T_n and the normal depth y_n. For this point, Eq. (8) gives:

$$B = \frac{T_n^2}{y_n} \tag{12}$$

where the subscript "n" indicates the uniform flow conditions.

The third point $E(T_0/2\,, y_0)$ which translate the fact that the top width T_0 is equal to the depth y_0. This special case is one and only for a given parabola. For a slender parabola the point E is located below the point P whereas for a widened or a much more opened parabola, the point E is located above the point P.

2.1.1 Characteristics of Uniform Flow in Parabolic Section

The uniform flow in parabolic channel (see Fig. 1) is characterized by:

1. The area of the wetted section A_n(m^2) of the channel is written:

$$A_n = 2\left[y_n \frac{T_n}{2} - \int_0^{T_n/2} Y dX\right] = \frac{2}{3} T_n y_n \tag{13}$$

2. The wetted perimeter P_n (m) is obtained by integrating length ds, of the parabola given by

$$P_n = \int ds = \sqrt{(dX)^2 + (dY)^2} =$$

$$P_n = \frac{T_n}{2}\left[\sqrt{1 + \left(\frac{4y_n}{T_n}\right)^2} + \frac{T}{4y_n} ln\left(\frac{4y_n}{T_n} + \sqrt{1 + \left(\frac{4y_n}{T_n}\right)^2}\right)\right] \tag{14}$$

3. The top width at water level

$$T_n = 4y_n z \tag{15}$$

4. Assume λ_n as the aspect ratio of the normal water area, defined by:

$$\lambda_n = y_n / T_n \tag{16}$$

With the aid of Eqs. (12) and (16), λ_n can be expressed as,

$$\lambda_n = \sqrt{y_n / B} \tag{17}$$

From Eq. (17), the normal depth y_n is thus:

$$y_n = B\lambda_n^2 \tag{18}$$

Keeping these geometric considerations in mind, Eqs. (13)-(14) and (15) can be rewritten respectively as:

$$P_n = \frac{B}{8}\left[4\lambda_n\sqrt{1 + 16\lambda_n^2} + ln\left(4\lambda_n + \sqrt{1 + 16\lambda_n^2}\right)\right] \qquad (19);$$

$$A_n = \frac{2}{3} B^2 \lambda_n^3 \tag{20}$$

and

$$T_n = B\lambda_n \tag{21}$$

The hydraulic radius $R_{h,n} = A_n / P_n$ is then:

$$R_h = B\delta(\lambda_n) \tag{22}$$

Eq. (22) gives the hydraulic radius $R_{h,n}$ as a function of the aspect ratio λ_n and the parameter B where:

$$\delta(\lambda_n) = \frac{16}{3} \frac{\lambda_n^3}{4\lambda_n\sqrt{1 + 16\lambda_n^2} + ln\left(4\lambda_n + \sqrt{1 + 16\lambda_n^2}\right)} \tag{23}$$

2.2. Chezy's Resistance Coefficient

2.2.1 General Expression

For uniform flow the Chezy's relation gives the discharge Q as:

$$Q = CA_n\sqrt{R_{h,n}I_f} \tag{24}$$

The aim of this work is to express the resistance coefficient of Eq. (24) using the RMM. The coefficient C, in addition to depending on the aspect ratio λ_n, also depends on other hydraulic parameters, such as the flow discharge Q, the longitudinal bed slope I_f, the absolute roughness ε of the internal wall of the channel, and the kinematic viscosity ν of the liquid. To do this, relationship (25) can be used to give the resistance coefficient C, established by [23] for turbulent flow for all geometric profiles of pipes and channels:

$$Q = -4\sqrt{2g}A_n\sqrt{R_{h,n}I_f}log\left(\frac{(\varepsilon/4R_{h,n})}{3.71} + \frac{10.04}{R_e}\right) \tag{25}$$

Where R_e is the Reynolds number, which may be defined by:

$$R_e = 32\sqrt{2}\frac{\sqrt{gI_fR_{h,n}^3}}{\nu} \tag{26}$$

By Eqs. (24) and (25), C can be given as

$$C = -4\sqrt{2g}log\left(\frac{(\varepsilon/4R_{h,n})}{3.71} + \frac{10.04}{R_e}\right) \tag{27}$$

It would appear from the Eq. (27) that C depends on ε, $R_{h,n}$, and R_e, which moreover to Eq. (26) depends on hydraulic radius $R_{h,n}$, the bed slope I_f, and the kinematic viscosity ν. In dimensionless terms, Eq. (27) becomes

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(\frac{(\varepsilon/4R_{h,n})}{3.71} + \frac{10.04}{R_e}\right) \tag{28}$$

Eq. (28) are applicable for all values of $R_e > 2300$ and for the wide range: $0 \leq \varepsilon/R_{h,n} \leq 0.2$. Inserting Eq. (22) into Eq. (26) results in:

$$R_e = 32\sqrt{2}[\delta(\lambda_n)]^{3/2}\frac{\sqrt{gI_fB^3}}{\nu} \tag{29}$$

Equation (26) can be rewritten as follows:

$$R_e = R_e^*[\delta(\lambda_n)]^{3/2} \tag{30}$$

Where R_e^* is a modified Reynolds number and is written by:

$$R_e^* = 32\sqrt{2}\frac{\sqrt{gI_fB^3}}{\nu} \tag{31}$$

According to Eqs. (22)-(30), the relation (28) can be rewritten as follows:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(\frac{\varepsilon/B}{14.8\delta(\lambda_n)} + \frac{10.04}{R_e^*[\delta(\lambda_n)]^{\frac{3}{2}}}\right) \quad (32)$$

It thus appears that C depends on the relative roughness ε/B, the aspect ratio λ_n and the Reynolds number R_e^*. When these parameters are given, relation (32) allows the explicit determination of the coefficient C. However, when it comes to design the channel, B is not a given data and only Q, λ_n, I_f, ε and $\nu$are the known parameters. In this case, Eq. (32) does not allow determining explicitly the coefficient C. However, this problem can be solved using the rough model method (RMM).

2.2.2 Calculation of Chezy's Resistance Coefficient Using the Rough Model Method (RMM)

All geometric and hydraulic characteristics of the rough model are distinguished by the symbol "$\overline{\ }$". Figure. 1 compares the geometric and hydraulic characteristics of the current channel with those of its rough model [33]. The rough model is particularly characterized by $\bar{\varepsilon}/\overline{D_h} = 0.037$ as the arbitrarily assigned relative roughness value, where $\overline{D_h}$ is the hydraulic diameter. The chosen relative roughness value is so large that the prevailed flow regime is fully rough. Thus, the friction factor is $\bar{f} = 1/16$ according to Eq. (7) for $R_e = \overline{R_e}$ tending to infinitely large value. Then, the Chezy's resistance coefficient $\bar{C}$ can be written as:

$$\bar{C} = \sqrt{\frac{8g}{\bar{f}}} = 8\sqrt{2}\sqrt{g} = \text{constant} \quad (33)$$

In this study there are two cases to calculate the Chezy's resistance coefficient according to the available data:

2.2.2.1 The Aspect Ratio λ_n of the Water Area is known

In the rough model, the channel is distinct by the dimension $\bar{B}$of the cross-section, the discharge $\bar{Q}$, a longitudinal slope $\overline{I_f}$, liquid kinematic viscosity $\bar{\nu}$ and a aspect ratio $\overline{\lambda_n}$. Hence our model is governed by the following conditions: $\bar{B} \neq B$; $\bar{Q} = Q$; $\overline{I_f} = I_f$; $\overline{\lambda_n} = \lambda_n$ and $\bar{\nu} = \nu$.
Using Eqs. (20) and (22) at a uniform flow, Eq. (24) becomes:

$$Q = \frac{2}{3}\lambda_n^3\delta(\lambda_n)^{1/2}\sqrt{C^2B^5I_f} \quad (34)$$

We put;

$$Q^* = \frac{2}{3}\lambda_n^3\delta(\lambda_n)^{1/2} \quad (35)$$

so that

$$Q^* = \frac{Q}{\sqrt{C^2B^5I_f}} \quad (36)$$

In accordance with formula (36), the relative conductivity of the rough model is defined by

$$\bar{Q}^* = \frac{Q}{\sqrt{\bar{C}^2\bar{B}^5I_f}} \quad (37)$$

By applying formula (33), Eq. (37) becomes:

$$\bar{Q}^* = \frac{Q}{\sqrt{128 g \bar{B}^5 I_f}} \tag{38}$$

By equalization of Eqs. (35) and (38) we get

$$\frac{Q}{\sqrt{128 g \bar{B}^5 I_f}} = \frac{2}{3}\lambda_n^3 \delta(\lambda_n)^{1/2} \tag{39}$$

As a result, we obtain:

$$\bar{B} = \left(\frac{3Q}{16\sqrt{2gI_f}}\right)^{0.4} \lambda_n{}^{-1.2}\delta(\lambda_n)^{-0.2} \tag{40}$$

Eq. (40) permits a direct determination of the parameter $\bar{B}$, since Q, I_f, λ_n and g are the known parameters of theproblem. Thus, all relationships are established for the explicit determination of the Chezy's coefficient C.

Further, using Eq. (29), the Reynolds number characterizing the flow in the rough model is:

$$\overline{R_e} = 32\sqrt{2}[\delta(\lambda_n)]^{3/2}\frac{\sqrt{gI_f\bar{B}^3}}{\nu} \tag{41}$$

$$\overline{R_e} = \overline{R_e^*}[\delta(\lambda_n)]^{3/2} \tag{42}$$

And

$$\overline{R_e^*} = 32\sqrt{2}\frac{\sqrt{gI_f\bar{B}^3}}{\nu} \tag{43}$$

According to the RMM [23], Chezy's coefficient C is related to the non-dimensional corrector factor of linear dimension ψ by the following simple equation:

$$C = \frac{\bar{C}}{\psi^{5/2}} \tag{44}$$

such as $0 < \psi < 1$. The non-dimensional correction factor of linear dimension ψ is related to the hydraulic characteristics of the rough model by the following relationship [23,34]:

$$\psi = 1.35\left[-log\left(\frac{\varepsilon/\overline{R_{h,n}}}{19} + \frac{8.5}{\overline{R_e}}\right)\right]^{-2/5} \tag{45}$$

Where $\overline{R_{h,n}}$ and $\overline{R_e}$ are respectively the hydraulic radius and the Reynolds number in the rough model.
Inserting Eqs. (22) and (42) into Eq. (45), leads to:

$$\psi = 1.35\left[-log\left(\frac{\varepsilon/\bar{B}}{19\delta(\lambda)} + \frac{8.5}{\overline{R_e^*}[\delta(\lambda_n)]^{3/2}}\right)\right]^{-2/5} \quad (46)$$

From Eqs. (44)-(33) and (46), one can write:

$$C = -5.343\sqrt{g}log\left(\frac{\varepsilon/\bar{B}}{19\delta(\lambda)} + \frac{8.5}{\overline{R_e^*}[\delta(\lambda_n)]^{\frac{3}{2}}}\right) \quad (47)$$

In dimensionless form, Eq. (39) can be rewritten as follows:

$$\frac{C}{\sqrt{g}} = -5.343log\left(\frac{\varepsilon/\bar{B}}{19\delta(\lambda_n)} + \frac{8.5}{\overline{R_e^*}[\delta(\lambda_n)]^{3/2}}\right) \quad (48)$$

Eq. (48) will be used when the parameter B of the channel is not a given data of the problem.

The coefficient C is explicitly calculated provided the discharge Q, the slope I_f, the absolute roughness ε and the aspect ratio λ_n are given.

To design the current channel, it is sufficient to calculate the linear dimension B using the following relationship, which applies to the entire turbulent flow field:

$$B = \psi\bar{B} \quad (49)$$

2.2.2.1.1 Practical Example 1

For the following data, compute Chezy's resistance coefficient in parabolic channel using the rough model method (RMM):

$Q = 0.915\ \mathrm{m^3/s}$; $I_f = 4 \times 10^{-4}$; $\varepsilon = 10^{-4}$ m ; $\lambda_n = 0.8$ and $\nu = 10^{-6}\ \mathrm{m^2/s}$.

Solution

1. For $\lambda_n = 0.8$, Eq. (23) can give $\delta(\lambda_n)$:
$\delta(\lambda_n = 0.8) = 0.21657831$
2. In accordance with the relationship (40), the parameter $\bar{B}$ of the rough model is:

$$\bar{B} = \left(\frac{3Q}{16\sqrt{2gI_f}}\right)^{0.4}\lambda_n^{-1.2}\delta(\lambda_n)^{-0.2} = \left(\frac{3 \times 0.915}{16\sqrt{2 \times 9.81 \times 4 \times 10^{-4}}}\right)^{0.4} 0.8^{-1.2} \times 0.21657831^{-0.2}$$
$$\bar{B} = 1.768862828\, m$$

3. Applying Eq. (43), the Reynolds number $\overline{R_e^*}$ is then :

$$\overline{R_e^*} = 32\sqrt{2}\frac{\sqrt{gI_f\bar{B}^3}}{\nu} = 32\sqrt{2}\frac{\sqrt{9.81 \times 4 \times 10^{-4} \times 1.768862828^3}}{10^{-6}}$$
$$\overline{R_e^*} = 6669153.428$$

4. Finally, according to Eq. (47), the Chezy's resistance coefficient C is:

$$C = -5.343\sqrt{g}log\left(\frac{\varepsilon/\bar{B}}{19\delta(\lambda)} + \frac{8.5}{\overline{R_e^*}[\delta(\lambda_n)]^{\frac{3}{2}}}\right)$$
$$C = -5.343\sqrt{9.81}log\left(\frac{10^{-4}/1.768862828}{19 \times 0.21657831} + \frac{8.5}{6669153.428 \times 0.21657831^{\frac{3}{2}}}\right)$$
$$C = 76.623\,\mathrm{m^{0.5}}/s$$

2.2.2.2 The Aspect Ratio λ_n of The Water Area is unknown

When the parameter λ_n is unknown, the following study shows a simplified technique for calculating Chezy's resistance coefficient, using the following parameters: the discharge Q, the parameter B, the bed slope I_f, the absolute roughness ε, and the kinematic viscosity ν. In practice, it is simple to measure each of these factors. In comparison to the strategy outlined in section 2.2.2.1, this simpler approach, which is likewise predicated on the theory of the rough model, results in a maximum relative deviation of roughly 1.25% [23,34]. In actuality, the relative inaccuracy used to measure absolute roughness is more than this relative deviation. Assuming $\lambda_n \neq \overline{\lambda_n}$ and applying Eq. (39) for the rough model leads to writing the following relationship:

$$Q^* = \frac{\overline{\lambda_n}^{4.5}}{\sqrt{4\overline{\lambda_n}\sqrt{1+16\overline{\lambda_n}^2} + ln\left(4\overline{\lambda_n} + \sqrt{1+16\overline{\lambda_n}^2}\right)}} \tag{50}$$

Where Q^* the relative conductivity is expressed as follows, according to Eq. (38):

$$Q^* = \frac{3\sqrt{3}Q}{8\sqrt{128gB^5I_f}} \tag{51}$$

All the parameters of Eq. (51) are known, which allows determining the value of the relative conductivity Q^*. What is needed is the computation of the aspect ratio $\overline{\lambda_n}$ using equation (50) for the given value of Q^*.
The aspect ratio $\overline{\lambda_n}(Q^*)$ of the implicit relation (50) can be reasonably written as the following power law [24]:

$$\overline{\lambda_n} = \alpha Q^{*\gamma} \tag{52}$$

Table 1: Values of α and γ for computation of the aspect ratio $\overline{\lambda_n}$ by Eq. (52)

Q^*	$\overline{\lambda_n}$	α	γ	Maximum deviation %
$Q^* \leq 0.00261$	$\overline{\lambda_n} \leq 0.30$	1.366	0.255	0.26
$0.00261 \leq Q^* \leq 0.0182$	$0.30 \leq \overline{\lambda_n} \leq 0.50$	1.441	0.264	0.15
$0.0182 \leq Q^* \leq 0.232$	$0.50 \leq \overline{\lambda_n} \leq 1$	1.487	0.272	0.26
$0.232 \leq Q^* \leq 69.5$	$1 \leq \overline{\lambda_n} \leq 5$	1.507	0.282	0.32

The computation of the aspect ratio $\overline{\lambda_n}$ by Eq. (52) allows the calculation of the Chezy coefficient according to the following steps:

1. Compute the non-dimensional correction factor of linear dimension $\psi(\overline{\lambda_n})$ by applying the explicit Eq. (45);
2. According to equation (33) with Eq. (44), the required value of the Chezy's coefficient is:

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} \tag{53}$$

3. Assign to the rough model the new linear dimension $B = \overline{B}/\psi$ according to Eq. (49) and derive the corresponding value of the relative conductivity Q^* using Eq. (51).

2.2.2.2.1 Practical Example 2

Computation the Chezy's resistance coefficient from the following data:
$Q = 8.0\ \mathrm{m^3/s}$; $I_f = 10^{-3}$; $T_m = 6$ m ; $Y_m = 3$ m; and $\nu = 10^{-6}\ \mathrm{m^2/s}$; $\varepsilon \to 0.00$

Solution

1. The linear dimension B is:

$B = T_m^2/Y_m = 6^2/4 = 12.0\ m$

2. Applying Eq. (51), the relative conductivity Q^* is then:

$$Q^* = \frac{3\sqrt{3}Q}{8\sqrt{128gB^5I_f}} = \frac{3\sqrt{3}\times 8}{8\sqrt{128\times 9.81\times 12^5\times 10^{-3}}} = 0.009295853$$

2. According to Eq. (52) and Table 1, the aspect ratio $\overline{\lambda_n}$ in the rough model is:

$$\overline{\lambda_n} = 1.441\times 0.009295853^{0.264} = 0.419075575$$

3. The hydraulic radius and Reynolds number are easily calculated in the rough model using Eqs. (22) and (41) respectively as follows:

$$\overline{R_{h,n}}(\overline{\lambda_n} = 0.419075575) = B\delta(\overline{\lambda_n}) = \frac{16B}{3}\left[\frac{\overline{\lambda_n}^3}{4\overline{\lambda_n}\sqrt{1+16\overline{\lambda_n}^2} + ln\left(4\overline{\lambda_n} + \sqrt{1+16\overline{\lambda_n}^2}\right)}\right]$$

$\overline{R_{h,n}} = 1.032810841\ m;$

4. Using Eq. (45), the non-dimensional correction factor of linear dimension ψ was easily calculated as:

$$\psi = 1.35\left[-log\left(\frac{\varepsilon/\overline{R_{h,n}}}{19} + \frac{8.5}{\overline{\overline{R_e}}}\right)\right]^{-\frac{2}{5}} = 0.67092637$$

5. According to the rough model method, the Chézy's resistance coefficient C is related to ψ by the Eq. (53) formula:

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} = \frac{8\sqrt{2\times 9.81}}{0.67092637^{5/2}} = 96.106405\ m^{0.5}/s$$

The value of the Chezy's resistance coefficient appears to be high, which is due to the presence of flow in the smooth turbulent region.

6. To determine the aspect ratio λ_n of the current channel, follow these steps:

7.1. According Eq.(49) assign to the rough model the following new value of linear dimension and derive the corresponding value of the relative conductivity Q^* using Eq. (51):

$$\bar{B} = B/\psi = \frac{12}{0.67092637} = 17.88571822\ m$$

And;

$$Q^* = \frac{3\sqrt{3}Q}{8\sqrt{128g\bar{B}^5I_f}} = \frac{3\sqrt{3}\times 8}{8\sqrt{128\times 9.81\times 17.88571822^5\times 10^{-3}}} = 0.003427491$$

6.2. Introducing this value of Q^*into Eq. (52), we obtain the aspect ration $\overline{\lambda_n}$ in the rough model equal to the aspect ration λ_n in the current channel as:

$$\overline{\lambda_n} = \lambda_n = 1.441\times 0.003427491^{0.264} = 0.322031036$$

7. This step aims to verify the validity of the calculations by determining the discharge Q using Chezy's equation. The discharge so calculated should be equal to the discharge given in the problem statement. Therefore, the wetted cross-sectional area using Eq. (20) and the hydraulic diameter must be calculated using Eq. (22):

$$A_n = \frac{2}{3}B^2\lambda_n^3 = \frac{2}{3} \times 12^2 \times 0.322031036^3 = 3.20600666\, m^2$$

And;

$$R_h(\lambda_n = 0.322031036) = B\delta(\lambda_n) = B\frac{16}{3}\left(\frac{\lambda_n^3}{4\lambda_n\sqrt{1+16\lambda_n^2} + ln\left(4\lambda_n + \sqrt{1+16\lambda_n^2}\right)}\right)$$

$$R_h(\lambda_n = 0.322031036) = 0.6738637\, m$$

Chezy's equation expresses the discharge Q as:

$$Q = CA_n\sqrt{R_{h,n}I_f} = 96.106405 \times 3.20600666\sqrt{0.6738637 \times 10^{-3}} = 7.998394\ m^3/s$$

The discharge so calculated and that given in the problem statement are almost equal. The deviation between both is about 0.02% only, which clearly indicates the validity of the calculations.

3. Variation of Chezy's resistance Coefficient

3.1. General Relationship

According to Eq. (32), the Chezy's resistance coefficient$C$ depends on three dimensionless variables namely, the relative roughness ε/B, the filling rate λ_n of the parabolic channel and the Reynolds number R_e^*. Its graphical representation is not easy, but it can be shown, as an indication, its variation for a fixed value of the relative roughness ε/B. This has been performed for different values of ε/B and for Reynolds number R_e^* varying between 10^4 and 10^8.

Among all the obtained graphs, those of Figs. 2 and 3, are representative. Fig. 2 translates the variation of $C/\sqrt{g}$ versus the filling rate λ_n and the Reynolds number R_e^*, for the value ε/Bcorresponding to a smooth inner wall of the channel. Fig. 3 shows the variation of $C/\sqrt{g}$ Reynolds number R_e^*, for $\varepsilon/B = 0.05$ corresponding to a state of the rough inner wall of the channel.

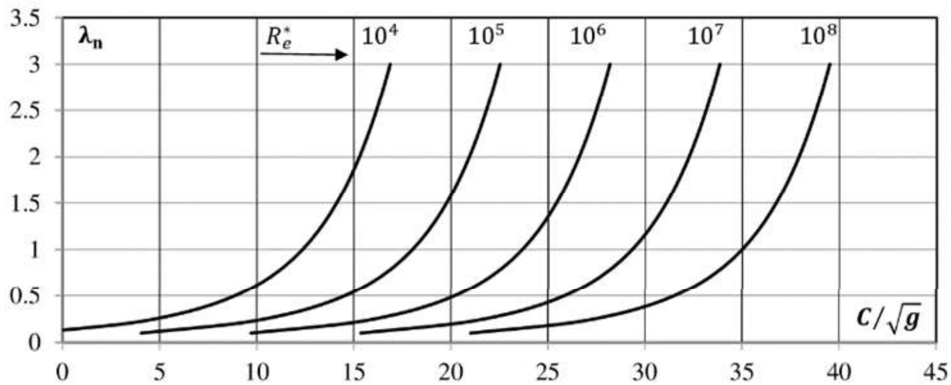

Figure 2: Variation of $C/\sqrt{g}$ versus λ_n and R_e^* according to Eq. (32), for $\varepsilon/B = 0.00$

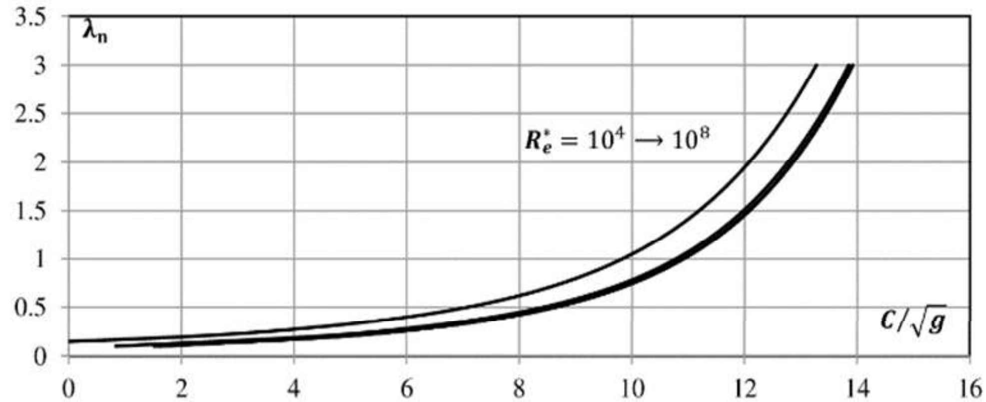

Figure 3: Variation of $C/\sqrt{g}$ versus λ_n and R_e^* according to Eq. (32), for $\varepsilon/B = 0.05$

For all curves, it is clear that when $0 < \lambda_n \leq 1$, the value of $C/\sqrt{g}$ increases rapidly, which is a characteristic for opened parabola, while when $\lambda_n \geq 1$ the value of $C/\sqrt{g}$ undergoes a very slow change in a wide range of λ_n independently of the value of the Reynolds number R_e^*, which is a characteristic for a slender parabola. This indicates that the channel shape also has a very large influence on the value of the Chezy coefficient. This important observation helps us design the channel under greatly improved flow conditions, when the aspect ratio λ_n is confined between zero and one.

When the roughness is zero ($\varepsilon/B = 0.00$), the $C/\sqrt{g}$ reaches higher values than those in case where the relative roughness is high. In this case, the dimensionless Chezy's resistance coefficient $C/\sqrt{g}$ is calculated according to the following relation:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(\frac{10.04}{R_e^*[\delta(\lambda_n)]^{\frac{3}{2}}}\right) \tag{54}$$

The relationship (54) shows that the rough model method is also applicable in the turbulent smooth regime. It also indicates that for the high chosen roughness value ($\varepsilon/B = 0.05$), the variation curves of $C/\sqrt{g}$ versus λ_n (see Fig. 3) are very close to each other and merge for the values $R_e^* \geq 10^5$.

This highlights the rough state of the flow, where $C/\sqrt{g}$ is almost independent of the Reynolds number R_e^* and depends solely on the value of the aspect ratio λ_n and the relative roughness ε/B of the channel. This case is governed by Eq. (29), writing that $R_e^* \longrightarrow +\infty$. Hence:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(\frac{\varepsilon/B}{14.8\delta(\lambda_n)}\right) \tag{55}$$

In order to obtain a minimum section and a uniform transverse resistance of the parabolic-shaped channel, the study conducted by Ohara and Yamatani [35] and Yang et al. [36] have demonstrated that the aspect ratio λ_n must be included in the interval]0;1] to improve the hydraulic performance and the swelling resistance of a parabolic-shaped channel. For the ultimate limit state where $\lambda_n = 1$, the explicit nature of relation (32) is obvious. The dimensionless Chezy's coefficient $C/\sqrt{g}$ can indeed be directly calculated from the known values of ε/B and R_e^* as follows:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(0.2355(\varepsilon/B) + 65.3212R_e^{*-1}\right) \tag{56}$$

Eq. (56) was established for the following approximate explicit solutions, depending on the relative roughness rang, according to relation (22), we have: $0.00 \leq \varepsilon/B \leq 0.0574$, and $R_e^* \geq 10^4$.

The study of Chey's resistance coefficient, based on the relationship (56), can reveal many details through the establishment of curves showing the variation of this coefficient as a function of relative roughness and Reynolds number. For the entire flow domain is in turbulent regime, this formula is satisfactory in practice and used as a best explicit approximation to evaluate $C/\sqrt{g}$ in terms of ε/B and R_e^*. It can be graphically represented in the semi-logarithmic division coordinate axis system in the Fig. 4 below.

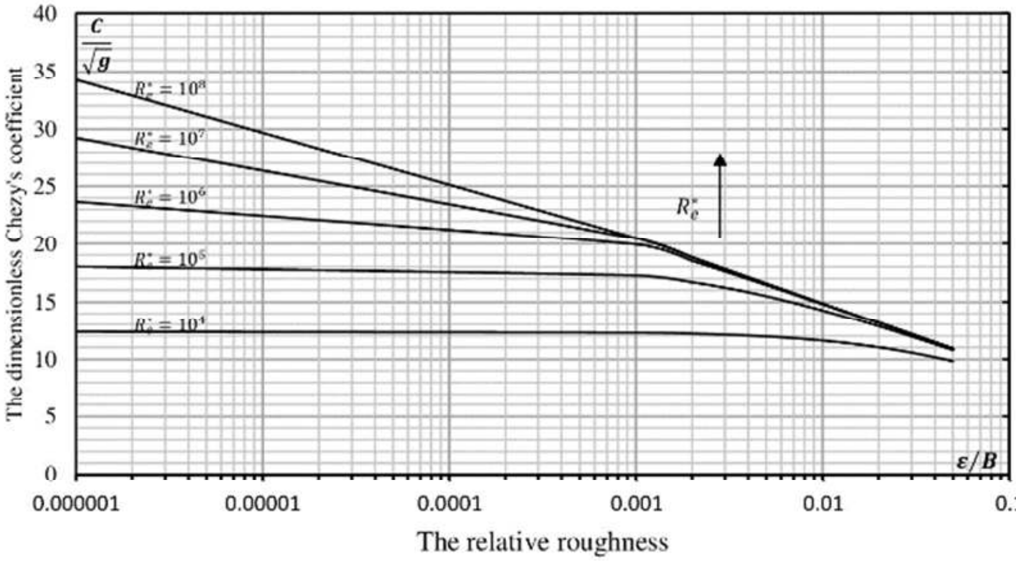

Figure 4: Variation of $C/\sqrt{g}$ versus (ε/B) according to Eq. (56) for various values of R_e^*

The diagram in the Fig. 4 shows that for a fixed value of the relative roughness ε/B, the dimensionless Chezy's coefficient $C/\sqrt{g}$ increases with the increase of the modified Reynolds number R_e^*. The obtained curves become more and more superimposed for high values of relative roughness ε/B. The obtained curves become closely overlapping for high values of relative roughness and are accompanied by an apparent decrease in the values of $C/\sqrt{g}$. Fig. 4 clearly shows the behaviour of flow resistance, expressed by the Chezy's coefficient in open channels, as a function of surface roughness with changes in the Reynolds number, which in turn includes the channel slope and the viscosity of the flowing fluid. On the other hand, the diagram in the Fig. 5 shows that for a fixed value of the relative roughness ε/B, the dimensionless Chezy's coefficient $C/\sqrt{g}$ increases with the increase of the modified Reynolds number R_e^*. The obtained curves become increasingly quasi-horizontal for high values of the relative roughness ε/B. Fig. 5 shows that dimensionless Chezy's coefficient $C/\sqrt{g}$ in the transition regime is lower than that of the rough flow, regardless of the value of the modified Reynolds number R_e^*.

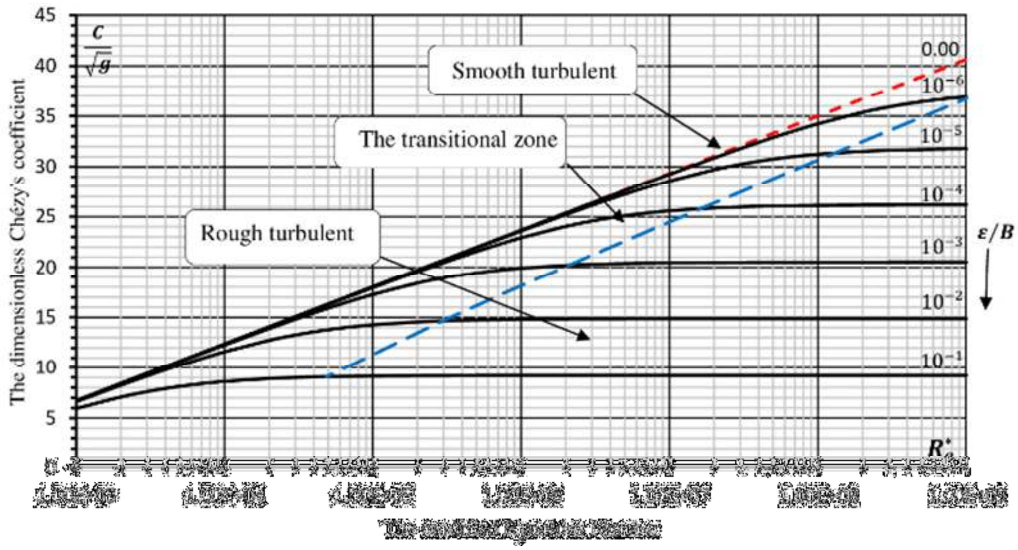

Figure 5: Variation of $C/\sqrt{g}$ versus R_e^* according to Eq. (56) for various values of (ε/B), (- - - -) limit curve separating the transition zone to the rough zone.

The plot of the diagram in Fig. 5 reveals, as in the case of the Moody diagram, the existence of three distinct zones. The first zone is reduced to a single curve (red dashed curve) corresponding to $(\varepsilon/B) = 0.00$. This is the flow in the smooth regime for which the variation $C/\sqrt{g}\ (R_e^*)$ corresponds to the relationship:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(65.3212{R_e^*}^{-1}\right) \tag{57}$$

The relationship (57) is also explicit with respect to the modified Reynolds number R_e^* and consequently the parameter B of the channel.

The second region is located between the red and blue dashed curves of the diagram in figure 5, and corresponds to the transition flow regime region.

The dimensionless Chezy's coefficient $C/\sqrt{g}$ depends on both the variables ε/B and R_e^*, according to the relation (56). For a fixed value of the relative roughness ε/B, the dimensionless Chezy's coefficient $C/\sqrt{g}$ increases as the modified Reynolds number R_e^* increases.

The third zone is located to the right of the diagram, and corresponds to the area of rough turbulent flow or hydraulically rough. The dimensionless Chezy's coefficient $C/\sqrt{g}$ in this area, is independent of the parameter R_e^*, this indicates that regardless of the kinematic viscosity ν of the flowing fluid, both the bed slope I_f and the channel dimension B**,** do not affect the value of the Chezy's coefficient. For a fixed relative roughness ε/B, the value of the parameter $C/\sqrt{g}$ remains constant up to a lower limit value of the parameter R_e^*. For a fixed relative roughness ε/B, the value of the parameter $C/\sqrt{g}$ remains constant up to a lower limit value of the parameter R_e^*, as indicated by the blue dashed curves, therefore, Eq. (56) is written as follows:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log\left(0.2355(\varepsilon/B)\right) \tag{58}$$

In Fig. 5, the limit curve is shown, separating the transition zone from the rough zone. The curve is represented by blue dotted lines and was drawn based on the approach of [37], by equalizing the values of $C/\sqrt{g}$ calculated via the tow relations (56) and (58) with a difference of 1.5%. This can be expressed as follows:

$$\left(\frac{C}{\sqrt{g}}\right)_{Rough\ zone} = 1.015 \times \left(\frac{C}{\sqrt{g}}\right)_{Transition\ zone} \tag{59}$$

So:

$$-4\sqrt{2}log\big(0.2355(\varepsilon/B)\big) = 1.015 \times \left[-4\sqrt{2}log\big(0.2355(\varepsilon/B) + 65.3212R_e^{*-1}\big)\right]$$

From where:

$$R_e^* = \frac{65.3212}{\left[\left(0.2355\frac{\varepsilon}{B}\right)^{1/1.015} - \left(0.2355\frac{\varepsilon}{B}\right)\right]} \tag{60}$$

The curve can be drawn by proceeding as follows:

1. Calculate the Reynolds number R_e^* by the expression (60) after having set the value of the relative roughness ε/B;
2. Then, the values of ε/B and R_e^* allow to calculate the $C/\sqrt{g}$ by the relationship (56).

Also, according to the relation (60) and through the calculations already made for the establishment of the limit curve, we can notice that for the rough turbulent flow regime, the Reynolds number is such that:

$$R_e^* \geq \frac{65.3212}{\left[\left(0.2355\frac{\varepsilon}{B}\right)^{1/1.015} - \left(0.2355\frac{\varepsilon}{B}\right)\right]} \tag{61}$$

However, for the transitional flow regime, the Reynolds number is as follows:

$$R_e^* < \frac{65.3212}{\left[\left(0.2355\frac{\varepsilon}{B}\right)^{1/1.015} - \left(0.2355\frac{\varepsilon}{B}\right)\right]} \tag{62}$$

3.1.1 Practical Example 3

Parabolic channel is defined by the linear dimensions: $T_m = Y_m = 2.5$ m, characterized by an absolute roughness $\varepsilon = 2 \times 10^{-4}$ m, kinematic viscosity liquid $\nu = 10^{-6}\ \mathrm{m}^2/s$, longitudinal slope $I_f = 2 \times 10^{-4}$ and $g = 9{,}81\ \mathrm{m}/\mathrm{s}^2$.

- Calculate the modified Reynolds number R_e^*, what is the regime flow?
- Calculate the Chezy's resistance coefficient C.

Solution

1. According to Eq. (11), the linear dimension B is:

$$B = T_m^2/Y_m = \frac{2.5^2}{2.5} = 2.5\ \mathrm{m}$$

2. The relative roughness:

$$\frac{\varepsilon}{B} = \frac{2 \times 10^{-4}}{2.5} = 8 \times 10^{-5}$$

3. From the expression (31):

$$R_e^* = 32\sqrt{2}\frac{\sqrt{gI_fB^3}}{\nu} = 32\sqrt{2}\frac{\sqrt{9.81 \times 0.0002 \times 2.5^3}}{10^{-6}} = 7923635.53$$

By relating the values of $\varepsilon/B = 8 \times 10^{-5}$ and $R_e^* = 7923635.53$ in Figure 5, we have a point on the transition zone. So the turbulent flow regime in the channel is transitional. And we can confirm that by the inequality (62):

$$R_e^* < \frac{65.3212}{\left[\left(0.2355\frac{\varepsilon}{B}\right)^{1/1.015} - \left(0.2355\frac{\varepsilon}{B}\right)\right]} = \frac{65.3212}{[(0.2355 \times 8 \times 10^{-5})^{1/1.015} - (0.2355 \times 8 \times 10^{-5})]}$$

$$R_e^* = 7923635.53 < 19877286.47$$

4. From the expression (56), the Chezy's resistance coefficient is calculated as follows:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}log(0.2355 \times 8 \times 10^{-5} + 65.3212 \times 7923635.53^{-1}) = 25.8365$$

Or;

$$C = 80.92\,\mathrm{m}^{0.5}/s$$

From the results of the above exercise, it is clear that the transition region defined by the dashed blue curve (see Fig. 4) is drawn with an excellent approximation to that determined by Hager [37].

4 Conclusions

The Chezy roughness coefficient often appears to be the most complete characteristic of hydraulic resistance to open flows in river channels comparing with other integral empirical characteristics of hydraulic resistance. In this paper, the Chezy's coefficient has been studied in detail. Using the general discharge relationship, the expression of the non-dimensional Chezy's coefficient $C/\sqrt{g}$ was established for a parabolic channel. The obtained expression clearly showed that $C/\sqrt{g}$ depends on the relative roughness ε/B, the aspect ratio λ_n of the water areaand the modified Reynolds number R_e^* characterizing the full state of the flow.This in turn depends on the bed slope I_f, the channel dimension $B$and the kinematic viscosity ν. All parameters affecting the flow in the expression $C/\sqrt{g}$ are represented by the relation (32), unlike the current relations. When the dimension$B$ of the channel is not a given data of the problem, the explicit calculation of the dimensionless Chezy's coefficient $C/\sqrt{g}$ is still possible through the use of the rough model method (RMM). $C/\sqrt{g}$ is then expressed as a function of the known parameters of the flow in the rough model by relation (48). In this case, the calculation of $C/\sqrt{g}$ requires the discharge Q, the bed slope I_f, the absolute roughnessε, the aspect ratio λ_n of the water area and the kinematic viscosity ν. When the user does not have the aspect ratio λ_n of the water area, the explicit calculation of the Chezy's resistance coefficient C is still possible thanks to the simplified method that was clearly described. This method uses the minimum measurable data in practice which are the discharge Q, the dimension B of the channel, the bed slope I_f, the absolute roughness ε and kinematic viscosity ν. This simplified method gives very satisfactory results.

The resulting relationship was presented in dimensionless terms, giving it a general validity character. From Eq. (32), curves are drawn in Figs. 2 and 3, its graphical representation shows that variation of $C/\sqrt{g}$ as a function of the aspect ratio λ_n, by assigning fixed values to the relative roughness $\varepsilon/B = 0.00$ and $\varepsilon/B = 0.05$, with the modified Reynolds number R_e^* increasing from 10^4 to 10^8. These curves also show that the Chezy resistance coefficient increases rapidly below the value of 1.0 of the aspect ratio λ_n with the increase in the modified Reynolds number and increases slowly above this value.

It's also clear that when the roughness of the inner walls of the channel is zero, the coefficient of resistance takes on higher values than when the roughness is high. This can be explained by the dominant effect of the Reynolds number induced by the viscosity ν of the liquid.

ln order to obtain a minimum cross-section and uniform transverse resistance of the parabolic channel, the research was supplemented by a particular study of the coefficient C where $\lambda_n = 1.0$. Then, the Eq. (56), expressing the Chezy's coefficient C, is derived from the Eq. (32). It is evidently applicable in all domains of the turbulent flow corresponding to $R_e^* \geq 10^4$ and $0.00 \leq \varepsilon/B \leq$

0.0574. Equation (56) is graphically represented using a semi-logarithmic axis of the coordinate system. This graphic shows that the Chezy` s resistance coefficient is not constant and may be observed in the rough, smooth, and transitional turbulent domains. This result permitted Chezy's formula to be generalized throughout the entire turbulent domain. Thus, a limit curve has been drawn separating the transitional flow regime or the rough flow regime. The location of both transition and rough zones was confirmed by the limitation of the Reynolds number value, respectively, through the inequalities (61) and (62).

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Ethics Approval: Not applicable.

Conflict of Interest: The author declares no conflicts of interest to report regarding the present study.

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